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Modelling Geotechnical Seismic Isolation Systems through the Preisach Formalism

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Abstract

Soil-structure interaction of structures protected by Geotechnical Seismic Isolation (GSI) system is investigated in this paper through a novel nonlinear approach. Preisach formalism is adopted to model the hysteretic behaviour of the soil through simplified nonlinear springs and dashpots. The efficiency of GSI versus natural soil is explored through extensive nonlinear numerical analyses on a benchmark structure. Comparisons with the currently adopted equivalent linear approach are also presented, highlighting the versatility of the proposed nonlinear model to reliably represent the nonlinear behaviour of the coupled structure-GSI system.

Keywords: nonlinear soil-structure interaction, rubber soil mixture, Preisach formalism, dynamic response, geotechnical seismic isolation, GSI

1 Introduction

Relatively recently, Geotechnical Seismic Isolation (GSI) has emerged as a technique to reduce the seismic vulnerability of structures [1]. Differently from common protection strategies, GSI does not directly modify the characteristics of the structure to be protected but works on the interface between the foundation of the structure and the surrounding terrain through the introduction of one or more layers of engineered soil. Since the protection concept is based on the modification of the soil-structure interaction, GSI requires a multidisciplinary approach, integrating insights from geology, civil engineering, and material sciences to design solutions that are both effective and economically viable.

Engineered soils used in GSI systems are developed by incorporating a variety of materials. Commonly used geosynthetics include geogrids, synthetic liners, geomembranes, and geotextile layers, as noted in [2] and [3]. Additionally, granular materials such as sand [4], gravel [5], stone pebbles [6], rubber-soil mixtures [7], and geofoam [8] are frequently utilized. In certain instances, hybrid and multi-layered solutions have also been explored, offering enhanced performance and adaptability [9],[10]. The properties of the engineered soils are tailored to achieve the desired isolation effect.

GSI systems mostly involve three main protection mechanisms for the main structure. The first uses the modified foundation soil to alter the system's dynamic response by shifting its fundamental frequency [11],[12]. This mechanism is usually realized either through a layer of low-modulus material or through a sliding interface, and its conceptual base is the same as the traditional base isolation mechanism. The second protection mechanism uses the dissipative properties of the engineered soil to reduce or filter the energy directed to the structure [13]. The third mechanism relies on deliberate sliding control between the structural foundation and the encompassing geomaterials [10].

The effectiveness of GSI has been extensively investigated, both through numerical simulations (see e.g. [1],[14]) and experimental research [15]-[17]. Notably, Pitilakis et al. [17] performed an experimental campaign on a full-scale prototype structure founded on gravel-rubber mixtures. Overall, GSI showed considerable potential in enhancing the seismic resilience of structures. Despite the large interest showed in the application of GSI systems, numerical studies are generally conducted mainly using either equivalent linear approaches see e.g. [14] or advanced non-linear FE modelling see e.g. [18]. In the last decade, however, a growing body of literature has been devoted to simplified models and the calibration of nonlinear springs able to capture the main features of nonlinear soil-structure interaction. In this regard, nonlinear rocking stiffness have been determined in [19] through an empirical approach based on FE analyses, further extended in [20] and applied to motorway bridge [21]. A nonlinear sway-rocking model has been developed [22] for shallow foundations. Li et al. [23] calibrated nonlinear translational and rotational springs through experimental data from centrifuge tests on pile foundations. Cacciola and Tombari [24] recently proposed the use of the Preisach formalism [25] to model the steady-state response of nonlinear soil structure interaction systems, further extended in [26] to the study of the seismic response of an existing historic bell tower.

In this paper, the efficiency of the Preisach formalism to model the seismic nonlinear soil-structure interaction of a structure protected by GSI is explored. Amplitude dependent equivalent springs and dashpots originally derived in [24] through a harmonic balance approach are calibrated in this study to model the behaviour of the structure resting on a GSI layer. Comparisons with the results from an equivalent linear analysis are presented and discussed in the paper.

2 Methods

Consider the building structure depicted in Figure 1a. The superstructure is considered to behave linearly. The foundation is assumed rigid and resting on a nonlinear geotechnical seismic isolation (GSI) system. The overall coupled behaviour is governed by the following equation of motion:

$$\boldsymbol{M}\ddot{\boldsymbol{u}}(t) + \boldsymbol{C}\dot{\boldsymbol{u}}(t) + \boldsymbol{K}\boldsymbol{u}(t) + \boldsymbol{h}(t) = -\boldsymbol{M}\boldsymbol{\tau}\ddot{u}_g(t) \qquad (1)$$

where $\boldsymbol{u}(t)$ is the vector collecting the i=1,…,n degrees of freedom of the system, u_i, $\boldsymbol{M}$ is the mass matrix, **C** is the damping matrix , $\boldsymbol{K}$ is the stiffness matrix, and $\boldsymbol{h}(t)$ is the nonlinear hysteretic vector encompassing the nonlinear forces generated by the soil structure interaction modelled as nonlinear springs, i.e. the horizontal force f_h and the moment f_θ, $\boldsymbol{\tau}$ is the influence vector and $\ddot{u}_g(t)$ is the ground acceleration.

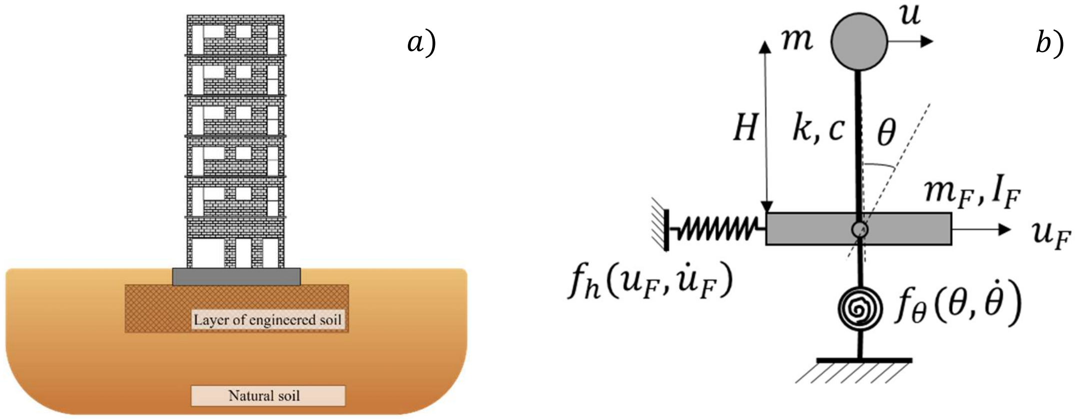

Figure 1: a) Structure resting over a non-linear compliant soil b) simplified mechanical 3DoF model

For illustrative purpose, consider the simplest case depicted in Figure 1b consisting of 3-DoF nonlinear soil-structure coupled system $(i.e.\ \boldsymbol{u}(t) = [u\ u_F\ \theta]^T)$. A similar model has been adopted in [14] for Structure-GSI interaction and in [24] for general soil-structure interaction. The pertinent matrixes listed in equation (1) reduce to

$$\boldsymbol{M} = \begin{bmatrix} m & 0 & 0 \\ 0 & m_F & 0 \\ 0 & 0 & I_F \end{bmatrix}, \qquad (2)$$

with m is the superstructure mass, m_F is the foundation mass, I_F is the foundation moment of inertia and

$$\boldsymbol{K} = \begin{bmatrix} k & -k & -k\,H \\ -k & k & k\,H \\ -k\,H & k\,H & k\,H^2 \end{bmatrix}. \tag{3}$$

In which k is the superstructure lateral stiffness and H the height of the superstructure. Also,

$$\boldsymbol{C} = \begin{bmatrix} c & -c & 0 \\ -c & c & 0 \\ 0 & 0 & 0 \end{bmatrix}, \tag{4}$$

with c the viscous damping of the superstructure. The hysteretic term can be written as

$$\boldsymbol{h}(t) = \begin{bmatrix} 0 & f_h(u_F, \dot{u}_F) + c_{Rh}\dot{u}_F & f_\theta(\theta, \dot{\theta}) + c_{R\theta}\dot{\theta} \end{bmatrix}^T, \tag{5}$$

where $f_h(u_F, \dot{u}_F)$ and $f_\theta(\theta, \dot{\theta})$ are the nonlinear hysteretic elements pertinent to the translational and rotational foundation degrees of freedom u_F and θ, while the terms $c_{Rh}\dot{u}_F$ and $c_{R\theta}\dot{\theta}$ have been herein introduced to account for energy dissipation due to radiation damping (see e.g. [14]). According to the Preisach formalism [15], the hysteresis is the result of the superposition of an infinite set of elementary hysteresis operators (hysterons or relay operators) $f_{\alpha,\beta}$, having local memory, that is

$$f_{i,H}(x, \dot{x}) = \iint_{\alpha \geq \beta} \mu(\alpha, \beta) f_{\alpha,\beta}(x, \dot{x}) d\alpha d\beta, \qquad i = h, \theta \text{ and } x = u_F, \theta, \tag{6}$$

where $\mu(\alpha, \beta)$ is an appropriate weight (or distribution) function that can be determined from experimental tests. Alternatively, suitable analytical functions (see e.g. [27-29]) can be adopted to represent various rheological models. In this regard, the simplest uniform distribution function $\mu(\alpha, \beta)$, been used in to capture the main features of soil-structure interaction problems [24]. It is noted that the implementation of equation (6) requires a bespoke numerical integration scheme updating local dominant maxima and dominant minima as described in [27]. As a simplification, using a harmonic balance approach [27-29] it is possible to determine closed form expressions of nonlinear equivalent damping and the equivalent stiffness. Namely, for the horizontal hysteretic element [24]

$$c_{e,h}(a_h) = \frac{a_h k_h^2}{3\pi\omega V_{max}}, \tag{7}$$

and

$$k_{e,h}(a_h) = k_h - \frac{k_h^2 a_h}{4V_{max}}; \tag{8}$$

while for the rotational element

$$c_{e,\theta}(a_\theta) = \frac{a_\theta k_\theta^2}{3\pi\omega M_{max}}, \tag{9}$$

and

$$k_{e,\theta}(a_\theta) = k_\theta - \frac{k_\theta^2 a_\theta}{4M_{max}} ; \tag{10}$$

where V_{max} and M_{max} assume the role of the limiting average horizontal load and average maximum attainable values of overturning moment of the foundation, respectively, while k_h and k_θ are the elastic stiffnesses of the given foundation. Also, in equations (7) and (9), ω is the circular frequency of the harmonic excitation that in the case of broadband excitation can be replaced by the equivalent natural frequency, amplitude dependent, of the system. As reported in [23] and [24], equations (7) and (9) might be conveniently rewritten as

$$c_{e,h}(a_h) = \eta_{e,h}(a_h)\sqrt{k_{e,h}(a_h)m} \tag{11}$$

and

$$c_{e,\theta}(a_\theta) = \eta_{e,\theta}(a_\theta)\sqrt{k_{e,\theta}(a_\theta)I_F} , \tag{12}$$

where $\eta_{e,h}(a_h)$ and $\eta_{e,\theta}(a_\theta)$ are the frequency independent loss factors given by [24]

$$\eta_{e,h}(a_h) = \frac{c_{e,h}(a_h)\omega}{k_{e,h}(a_h)} = \frac{4}{12\pi\frac{V_{max}}{k_h a_h} - 3\pi}; \quad \forall \frac{V_{max}}{k_h a_h} < \frac{1}{4} \tag{13}$$

and

$$\eta_{e,\theta}(a_\theta) = \frac{c_{e,\theta}(a_\theta)\omega}{k_{e,\theta}(a_\theta)} = \frac{4}{12\pi\frac{M_{max}}{k_\theta a_\theta} - 3\pi}; \quad \forall \frac{M_{max}}{k_\theta a_\theta} < \frac{1}{4} . \tag{14}$$

It is noted that the simplified expressions of the equivalent stiffnesses and dashpots are functions of the instantaneous peak amplitude of the foundation displacement a_h and rotation a_θ, while the full (and clearly more accurate) Preisach model of hysteresis takes into account the memory of the system through the dominant maxima and minima. Nevertheless, it has been shown ([24], [26]) that if appropriately calibrated, those nonlinear elements can reliably describe the evolution of the stiffness reduction and damping increment of structures undergoing base vibrations. As a consequence, the hysteretic term given in equation (5) is replaced by the equivalent nonlinear one

$$\boldsymbol{h}_{\mathrm{eq}}(t) = \left[0 \quad k_{e,h}(a_h)u_F + (c_{e,h}(a_h) + c_{Rh}+)\dot{u}_F \quad k_{e,\theta}(a_\theta)\theta + (c_{e,\theta}(a_\theta) + c_{R\theta})\dot{\theta}\right]^T \tag{15}$$

to explore the performance of GSI, accounting for its inherent nonlinear behavior.

3 Results

In this section, the benchmark model proposed in [14] and [24] (see Figure 1b) is investigated for the study of the efficiency of the geotechnical seismic isolation system made of rubber soil mixture (GSI-RSM) as a seismic protection tool through a nonlinear, albeit simplified, approach. The structure geometrical and mechanical parameters provided in [14] necessary for the model are reported in Table 1. The structure undergoes ground acceleration. The Northridge (1994) ground motion time-history scaled to 1g has been used first for the application to enable direct comparison with results from [14].

Data	Value
H [m] (height of the superstructure)	10
m [Mg] (mass of the superstructure)	650
m_F [Mg] (mass of the foundation)	260
I_F[Mg m^2]	5.62x10^3
T_{FB} [s] (fundamental period of the superstructure assumed fully fixed)	0.37
ζ (damping ratio of the superstructure)	0.05

Table 1: Mechanical and Geometrical Parameters of the structural model

The proposed equivalent nonlinear springs and dashpots (equations (7)-(10)) require the knowledge of V_{max} and M_{max} that in principle are derived from a nonlinear static analysis. For the purpose of the present application, they have been determined using the equivalent stiffness and damping provided in [14]. Tables 2 and 3 show the relevant soil and foundation equivalent parameters, pertinent to the natural soil and for the GSI-RSM, which were used for the calibration of the Presiach nonlinear elements.

Data	Value
$k_{h,EL}$ [kN/m] (equivalent stiffness)	304x10^3
c_{Rh} [kNm/s] (radiation damping)	25.3x10^3
$k_{\theta,EL}$ [kNm] (equivalent stiffness)	17.3x10^6
$c_{R\theta}$ [kNms] (radiation damping)	1110x10^3
ζ_{eq} (%) (equivalent damping ratio)	22.8
G/G_{max}	0.038

Table 2: Soil and Foundation mechanical parameters for Natural Soil [14]

Data	Value
$k_{h,EL}$ [kN/m] (equivalent stiffness)	87.7x10^3
c_{Rh} [kNm/s] (radiation damping)	7.27x10^3
$k_{\theta,EL}$ [kNm] (equivalent stiffness)	2.82x10^6
$c_{R\theta}$ [kNms] (radiation damping)	326x10^3
ζ_{eq} (%) (equivalent damping ratio)	28.7
G/G_{max}	0.086

Table 3: Soil and Foundation mechanical parameters for GSI-RSM [14]

Specifically, the following positions have been made for model calibr ation

$$k_{e,h}(a_{h,max}) = k_{h,EL}; \quad k_{e,\theta}(a_{\theta,max}) = k_{\theta,EL};$$
$$\eta_{e,h}(a_{h,max}) = \eta_{e,\theta}(a_{\theta,max}) = 2\,\zeta_{eq}. \qquad (16)$$

where $a_{h,max}$ and $a_{\theta,max}$ are the maximum foundation displacement and rotation, respectively. Those quantities are not known *a priori*, so they have been determined iteratively. Figures 2 show the comparison of relevant response parameters of the benchmark structure with and without the GSI-RSM system for both equivalent linear [14] and nonlinear models.

As the nonlinear springs have been calibrated to match the equivalent translational and rotational base stiffness, the responses overall match fairly well with the results from the equivalent linearization, highlighting the accuracy of the proposed nonlinear model. Minor differences can be observed at the initial segment of the time histories as the nonlinear stiffnesses (and dashpots) did not reach their target values.

Larger differences can be observed in the moment-base rotation loops shown in Figure 3, which highlights how the nonlinear model realistically follows the evolution of stiffness reduction until the convergence to the target values. It is noted that once the model has been calibrated imposing equations (16) the value of the limiting average horizontal load, V_{max} , and the average maximum attainable values of overturning moment, M_{max}, of the foundation are also determined leading respectively to the values: $V_{max} = 3.29 \times 10^6 N$ and : $M_{max} = 3.41 \times 10^7 Nm$, respectively. Those values have been used for model verification determining the response of the benchmark structure to a different ground motion time history, namely the Duzce, Turkey 1999 earthquake, without imposing equations (16). As it can be seen from Figures 5 and 6 the results are in excellent agreement with those determined in [14] manifesting the robustness of the proposed nonlinear spring model.

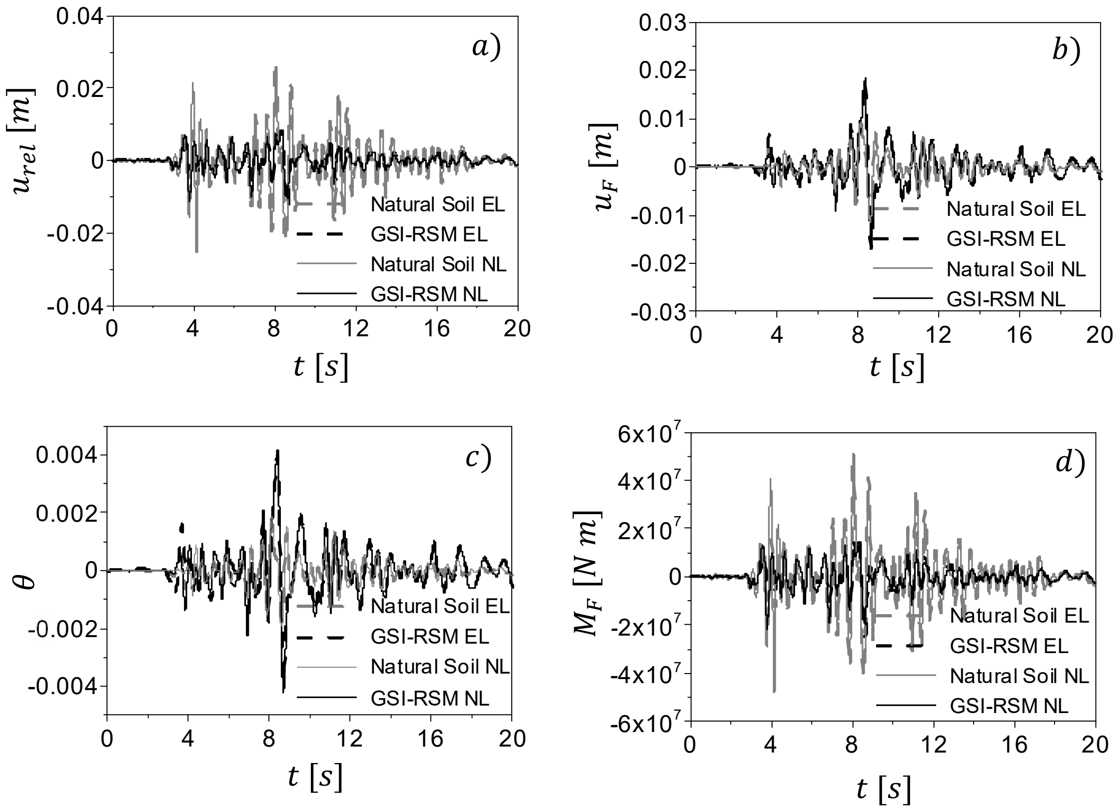

Figure 2: Comparison between the response of the benchmark structure forced by the Northridge 1994 earthquake with and without the GSI-RSM system using the Equivalent Linearization (EL) and the simplified Nonlinear approach: a) relative displacement, b) base displacement, c) base rotation and d) base moment

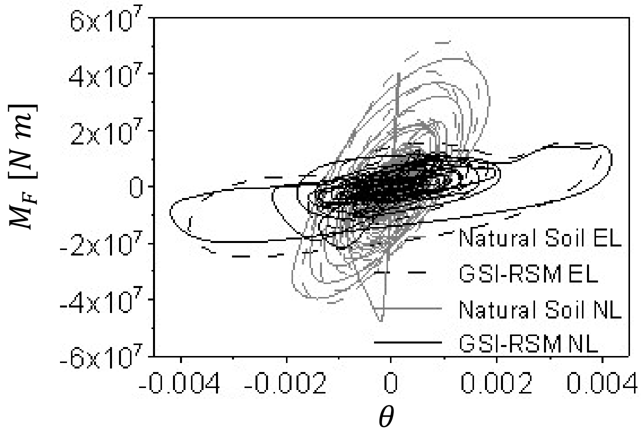

Figure 3: Comparison between the moment-base rotation loops with and without the GSI-RSM system using the Equivalent Linearization (EL) approach and the simplified Nonlinear (NL)

4 Conclusions and Contributions

In this paper, the structure-GSI interaction is explored through equivalent nonlinear springs and dashpots determined through the Preisach formalism. Closed form solutions have been determined using a harmonic balance approach leading to nonlinear stiffness and dashpot dependent on the amplitudes of the base translation and rotation. To highlight the versatility of the proposed nonlinear elements a simple calibration has been performed to match the equivalent foundation stiffness and damping determined in literature for a benchmark structure. The results are in excellent agreement once reached the base response peak values manifesting, as expected, discrepancies for lower values of the base response amplitude. The robustness of the proposed model has been verified using a different ground motion time history without the necessity to recalibrate the nonlinear springs. Excellent matching highlighted the versatility of the model. Different results can clearly be observed for different calibrations using the limiting average horizontal load and average maximum attainable values of overturning moment of the foundation from pushover analysis of analytical formulations. Future works will focus on comparison with experimental data to highlight the importance of an accurate nonlinear model of the GSI system adopting the full hysteretic Preisach model.

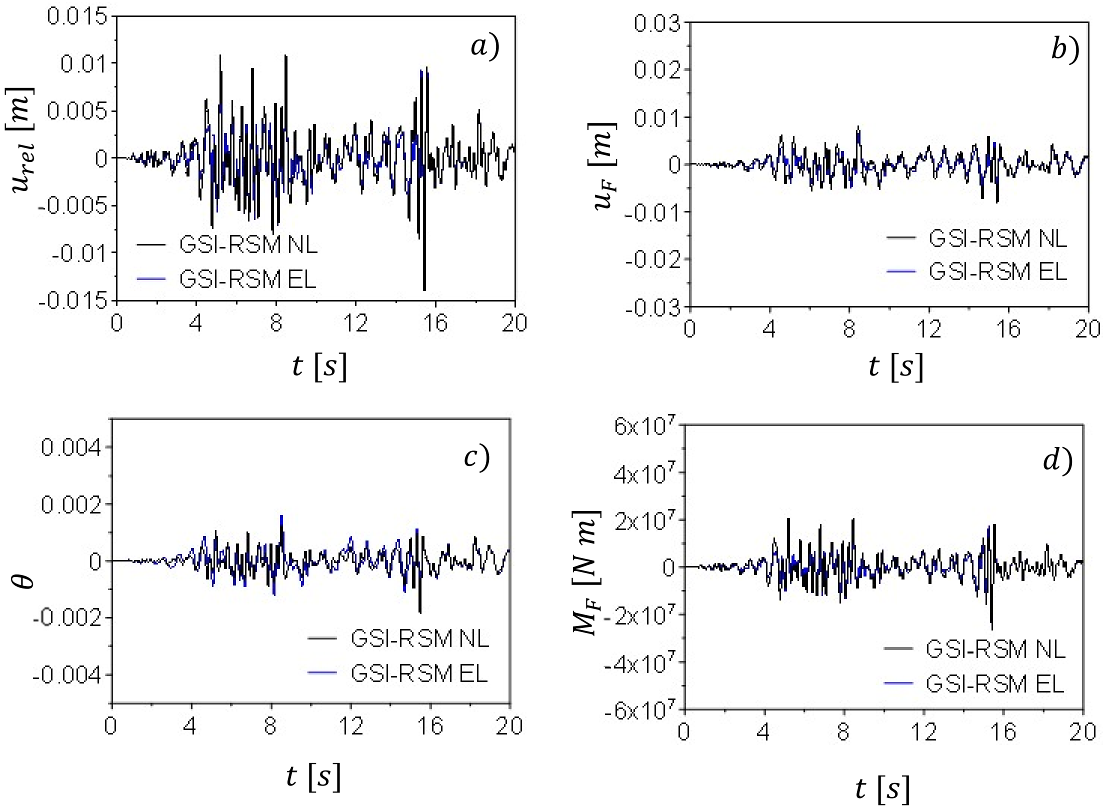

Figure 4: Comparison between the response of the benchmark structure forced by the Duzce, Turkey 1999 earthquake using the Equivalent Linearization (EL) approach and the simplified Nonlinear (NL) springs a) relative displacement, b) base displacement, c) base rotation and d) base moment

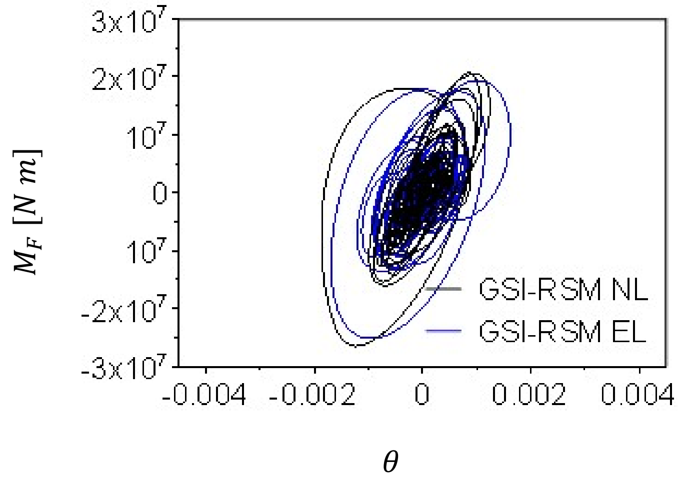

Figure 5: Comparison between the moment-base rotation loops of the benchmark structure forced by the Duzce, Turkey 1999 earthquake using the Equivalent Linearization (EL) and the simplified Nonlinear (NL) approach

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