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Finite Element Formulation for Buckling Analysis of Angle-Ply Beam-Type Structures Considering Shear Deformation Effects

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Abstract

This paper presents a novel shear deformable numerical model designed for the nonlinear stability analysis of beam-type structures. The incremental equilibrium equations are derived for a straight thin-walled beam element using an updated Lagrangian formulation. This formulation accounts for the nonlinear displacement field of cross-sections, considering both restrained warping and large rotation effects. The model incorporates shear deformation effects by addressing coupling effects such as bending-bending and bending-warping torsion in the composite cross-section. Cross-section properties are computed based on the reference modulus, enabling the modelling of various laminate configurations. A numerical algorithm is developed to determine the geometric properties of the composite cross-section. The proposed model is validated through examination of different material configurations and presentation of several benchmark examples. Results indicate that the model is devoid of shear locking issues and demonstrates reliable performance.

Keywords: thin-walled, composite cross-section, beam model, buckling, large displacement, nonlinear stability analysis.

1 Introduction

Load-bearing composite structures often feature slender beam elements with thin-walled cross-sections, introducing increased complexity and susceptibility to deformation instability and buckling in their response to external loading [1-3].

Various forms of instability, including pure flexural, pure torsional, torsional-flexural, or lateral deformation, may manifest in these optimized structures. Therefore, accurately determining the buckling strength, representing the limit state of deformation stability, is crucial during the design process.

While analytical solutions exist for simpler cases [4], the necessity for numerical solutions becomes apparent. Several studies [5-9] have presented geometric nonlinear analyses of composite beam structures, considering the influence of shear deformation. These works address bending-bending and bending-warping torsion coupling shear deformation effects, particularly in asymmetric cross-sections where the principal bending and shear axes do not align.

In a prior publication by the authors [9], composite frames were introduced using geometrically nonlinear beam elements that account for shear deformation effects, albeit focusing solely on cross-ply laminated composite structures. The current study aims to conduct a large displacement nonlinear analysis of thin-walled beam-type structures, considering shear deformation effects and incorporating material inhomogeneity in the form of angle-ply laminates. The analysis exclusively relies on the authors' developed numerical model, and the results will be compared with those obtained from relevant sources.

2 Methods

To incorporate shear deformation effects, this formulation relies on modified Timoshenko's theory for non-uniform bending and modified Vlasov's theory for non-uniform or warping torsion. Additionally, the present work introduces an enhanced shear-deformable beam formulation, addressing bending-bending and bending-warping torsion coupling shear deformation effects [5-10]. These effects become prominent in asymmetric cross-sections where the principal bending and shear axes do not coincide [11]. The beam member is assumed to be prismatic and straight, while external loads are treated as conservative and static.

The geometric stiffness of the element is derived using the updated Lagrangian (UL) incremental formulation [12,13], which incorporates the non-linear displacement field of the cross-section. This field includes second-order displacement terms to accommodate large rotation effects. Consequently, the incremental geometric potential of the semitangential moment is determined for internal bending and torsion moments, ensuring moment equilibrium conditions are maintained at the frame joint, where beam members with different spatial orientations are connected [14,15]. Through the use of cubic interpolation for deflections and twist rotation, along with an interdependent quadratic interpolation for slopes and the warping parameter, considering shear-deformable effects, a locking-free beam element, known as a super-convergent element, is obtained. This element eliminates the need for reduced integration techniques to prevent shear locking [16]. In terms of the incremental-iteration scheme, the generalized displacement control method [15] is employed, and nodal orientation updating at the end of each iteration is conducted using the transformation rule applicable to semitangential rotations [17,18]. The force recovery phase follows a conventional approach [18,19].

To address material inhomogeneity within a composite cross-section, a separate numerical model is employed for calculating cross-sectional properties. These properties are weighted by the reference modulus [11]. All cross-sectional properties are defined for the middle line of the cross-sectional branch and currently apply exclusively to balanced and symmetrical angle-ply laminates.

3 Results

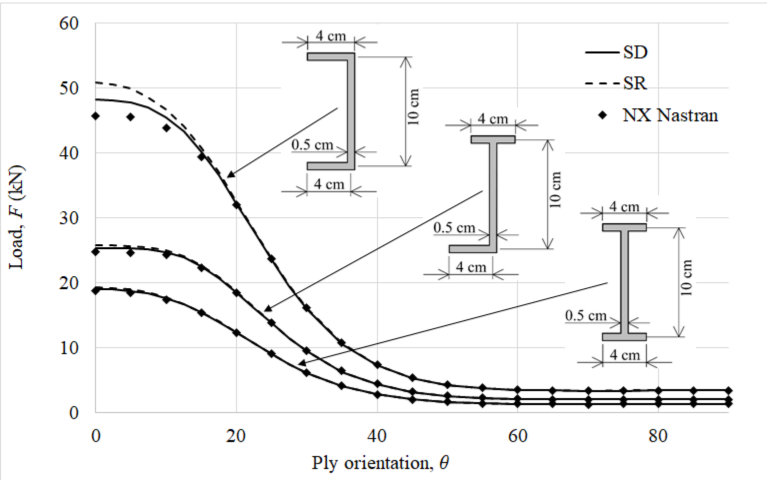

Figure 1: Cross-section shapes and buckling load versus ply orientation (θ) for the cantilever column.

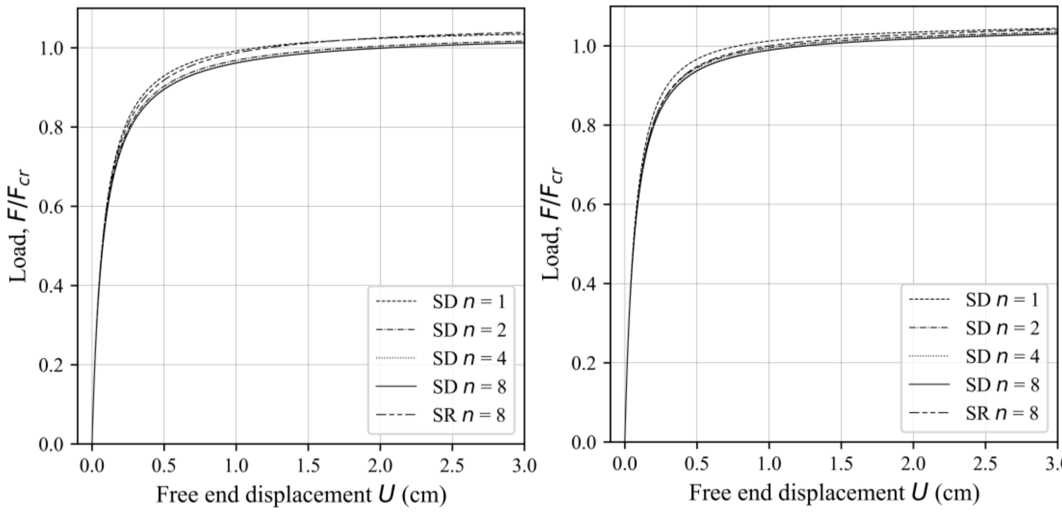

Figure 2: Convergence analysis for the cantilever column, prebuckling response for ply orientation $\theta = 10°$. Left: monosymmetric U profile. Right: asymmetric profile.

The THINWALL v.18 software has been developed using a finite element procedure that integrates various methods discussed in the preceding section. This program is capable of conducting both linearized and nonlinear stability analyses. For nonlinear stability analyses, the program employs the generalized displacement control method, introducing a small perturbation load to initiate buckling. This load-deformation approach enables a thorough investigation of stability issues, evaluating the structural behavior across the entire range of interest. It covers both pre- and post-buckling phases, providing a comprehensive plot of the structure's loading as a function of deformation. This approach offers more reliable information for actual structures and loading conditions compared to the eigenvalue approach.

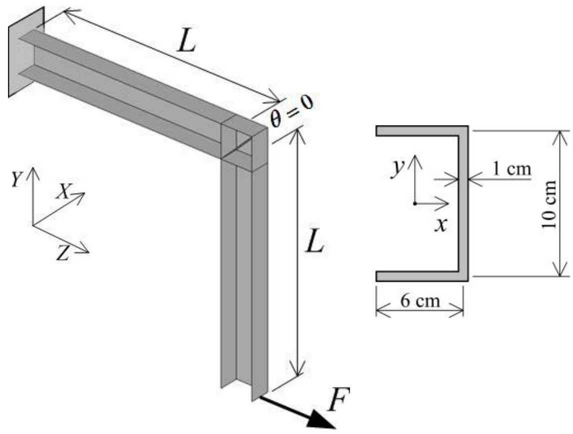

Figure 3: *L*-frame configuration under a load (*F*) in the *Z*-direction at the free end.

To evaluate the impact of shear deformability on the stability behaviour of the studied structural members, two model comparisons are conducted. The first comparison, identified as 'SR' in the presented results, neglects shear deformability effects entirely. The second model, referred to as 'SD' in the presented results, incorporates shear deformation effects using the methodology outlined in this paper. The material under examination is graphite-epoxy (AS4/3501), characterized by the following properties: longitudinal elastic modulus (E_1) = 144 GPa, transverse elastic modulus (E_2) = 9.65 GPa, shear modulus (G_{12}) = 4.14 GPa, and Poisson's ratio (ν_{12}) = 0.3.

In the initial example, a cantilever column with a length (L) of 100 cm, subjected to an axial force (F) is examined. The cross-section shapes being analysed are depicted in Figure 1. In the case of the asymmetric profile, the column experiences buckling in a torsional-flexural mode. For the other two profile shapes column experiences buckling in a flexural mode. Each branch of the cross-section consists of a symmetric and balanced laminate with a $[\theta/\text{-}\theta]_{2S}$ stacking sequence, where all plies share the same thickness. Figure 1 illustrates the correlation between the buckling load and the ply orientation (θ).

In the nonlinear stability analysis, a small perturbation force with an intensity of $0.001F$ is introduced, acting in the X-direction. Figure 2 presents the nonlinear convergence study for four different mesh configurations, including one, two, four, and eight beam elements. The results in Figure 2 are normalized by the critical buckling force obtained from the laminate shell solution using NX Nastran software.

For a ply orientation of $\theta = 10°$, the critical buckling force (F_{CR}) is 17.5 kN for the I profile, 43.9 kN for the monosymmetric U profile and 24.2 kN for the asymmetric profile. The SD model consistently and accurately identifies the buckling state across all mesh configurations used, while the SR model tends to overestimate the buckling strength by approximately 10%.

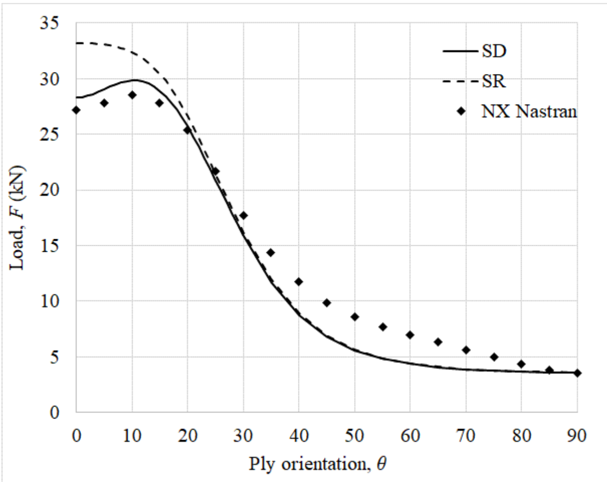

Figure 4: Buckling load versus ply orientation (θ) for the mono-symmetric channel profile in the L-frame.

In the second example, an L-frame is subjected to a load (F) in the Z-direction, applied through the centroid of the cross-section at the free end, as depicted in Figure 3. A mono-symmetric channel profile is utilized, and it is assumed that there is full warping restraint at the frame corner. In the case of the mono-symmetric channel profile, the frame experiences buckling in a lateral-torsional mode.

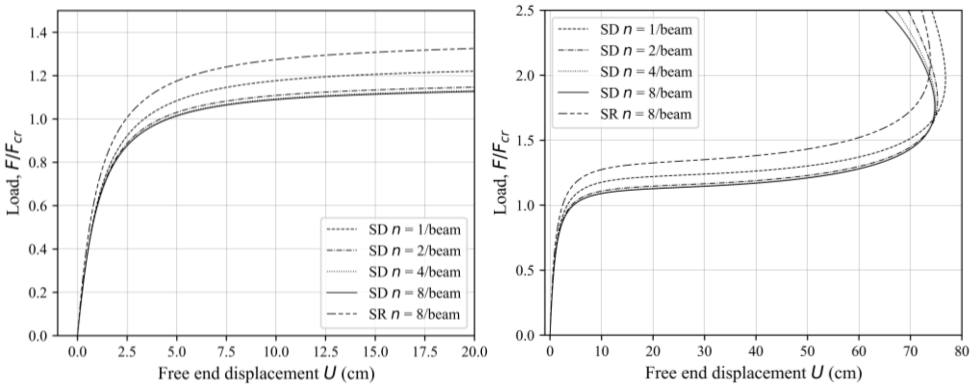

Figure 5: Convergence analysis for the L-frame ($\theta = 0°$). Left: Prebuckling response. Right: Postbuckling response.

In this specific case, each branch of the cross-section is built using a symmetric and balanced laminate with a $[\theta/\text{-}\theta]_{2S}$ stacking sequence, where all plies have the same thickness. Figure 4 illustrates the relationship between the buckling load and the ply orientation (θ). In this example, it is observed that the buckling strength reaches its maximum when the ply orientation is set at 10°. However, in this case, the effect of shear deformations is notably more pronounced, exerting a more substantial influence on the overall buckling behaviour.

In the nonlinear stability analysis, a small perturbation force with an intensity of $0.001F$ is applied in the X-direction. The results presented in Figures 5 and 6 are normalized by the critical buckling force obtained through the laminate shell solution from NX Nastran software. The critical buckling force for a ply orientation of $\theta = 0°$ is $F_{CR} = 25.1$ kN, and for a ply orientation of $\theta = 10°$, it is $F_{CR} = 26.3$ kN. Figures 5 and 6 depict the nonlinear convergence study for four different mesh configurations, each consisting of one, two, four, and eight beam elements per frame member. It is evident that the SD model consistently and accurately identifies the buckling state across all mesh configurations, whereas the SR model overestimates the buckling strength.

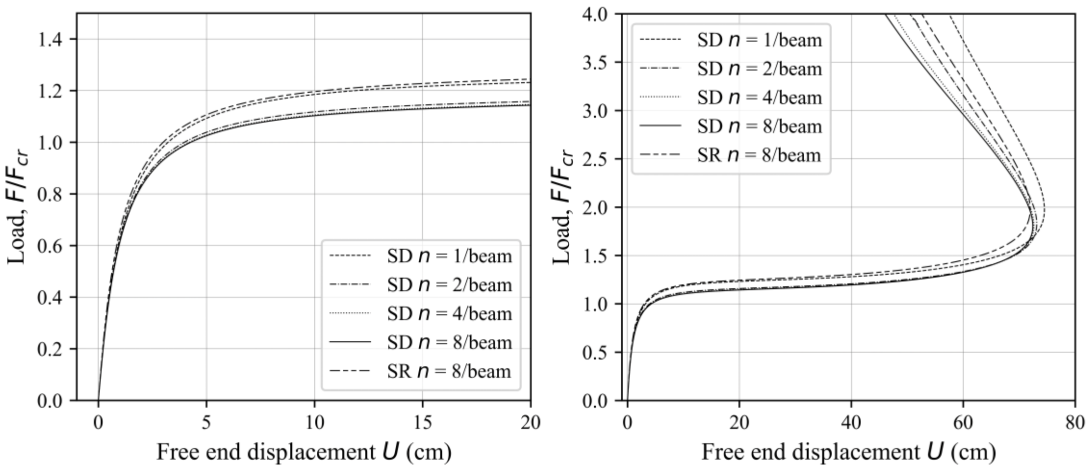

Figure 6: Convergence analysis for the L-frame ($\theta = 10°$). Left: Prebuckling response. Right: Postbuckling response.

4 Conclusions and Contributions

A refined shear-deformable beam formulation has been proposed for the geometrically nonlinear stability analysis of composite frames. This formulation takes into account the impact of shear deformation resulting from non-uniform bending and torsion in thin-walled beams with asymmetric cross-sections. Benchmark examples have been presented to validate the model's performance. The significance of including shear deformations in the formulation is apparent, as evidenced by a significant reduction in stability strength compared to models that neglect shear effects.

The developed model provides flexibility in handling warping restraints at nodes, offering control at either a global or local level. Global control involves implementing warping prevention for all beam elements connected at a common joint. On the other hand, local control can be achieved by using warping transformation parameters that indicate the specific warping restraint conditions for individual beam elements at a given node.

To validate the accuracy and reliability of the model, shear-locking tests were conducted with various mesh configurations. The results of these tests affirm that shear locking does not occur in the model, providing evidence of its effectiveness in accurately capturing shear deformation behaviour.

Moreover, the model has been employed to analyse the influence of various material configurations, showcasing and verifying their impact on the critical load through selected examples.

Acknowledgements

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