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Investigation of Beam Finite Element Models of Octet-Truss Unit Cell Using Homogenization

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Abstract

This study investigates the beam model accuracy of octet-truss lattice solid cell in the calculation of homogenized material properties using the average stress method with Ansys finite element program for a range of relative density 0.01-0.5. While alignment is observed at lower relative densities (less than 0.1), the beam model underestimates results at higher densities, revealing reductions in elastic and shear modulus values. The study critically examines existing beam model modifications in literature, traditionally centred on increasing strut stiffness at joint regions, particularly for lattice materials under compression testing. The studies found in the literature ultimately produced multiple parameter pairs by considering only the modulus of elasticity. The procedure for determining these parameters is graphically illustrated in detail considering also the shear modulus and it is concluded that both modulus values could not be fit to the solid model results with this type of modification. Closed expressions for the moduli of octet-truss lattice material are presented to be used for the relative density range higher than 0.1, to keep the error under ten percent.

Keywords: homogenization, average stress method, lattice material, octet-truss unit cell, beam finite element simulation, shear stiffness.

1 Introduction

Additive Manufacturing (AM) technology, finds extensive applications such as aerospace, automotive, architecture, biomedical, packaging, and sports industries. Particularly valued for its ability to deliver ultralight weight components with high energy absorption, effective heat isolation and dissipation, as well as elevated specific strength and stiffness. One of the notable advantages of AM is the capacity to tailor material properties by modifying the geometry of the material's structure, allowing for the precise design of desired material constants [1].

Cellular materials are classified into two major groups based on their cell structures, known as stochastic (non-periodic) and non-stochastic (periodic) [2]. Octet-truss (OT) is a strut-based non-stochastic open-cell lattice material which is a product of AM technology. OT unit cell is geometry is shown in Figure 1(a), where R is the radius and L is the length of struts. There are 36 struts of the same length. Joints are located at the face centers and at the corners. At each joint, 12 struts connect each other, resulting in a lattice connectivity of $Z = 12$. It is a stretching-dominated lattice structure based on Maxwell classification [3]. Stretch-dominated structures have higher modulus and the initial collapse strength compared to bending-dominated structures of the same relative density. This makes stretch-dominated structures well-suited for lightweight load-bearing applications. Extensive research has been conducted on the octet-truss lattice material due to its high specific properties [3-5]. The relative density of octet truss $\bar{\rho} = \rho/\rho_b$, is defined as the ratio of the macroscopic density of a cellular material ρ and the density of the bulk material ρ_b. That of the octet truss unit cell with cylindrical struts in terms of aspect ratio (R/L) is given in reference [3] as follows.

$$\bar{\rho} = 6\pi\sqrt{2}\left(\frac{R}{L}\right)^2 - \frac{32}{2-\sqrt{2}}\left(\frac{R}{L}\right)^3 \qquad (1)$$

Here, the first term represents the material volume occupied by all of the struts, with the exception of half of the struts located on cell faces, as they belong to the adjacent cells. The second term in Equation (1) is subtracted to exclude the overlapping volumes of the struts at the joints. This term becomes significant for high relative densities.

Homogenization methods are used to replace the detailed geometry of the lattice structure with an equivalent homogeneous medium for which the effective material properties are derived. Lattice structures are composed of repetition of numerous cells. The purpose of homogenization is to reduce time and effort spent on the large dimensioned structures which are made of these types of materials. On the other hand, micro model is the physically exact simulation of the material including all the material and geometry details such as different materials, voids, interface properties, etc. In order to characterize the material, tensile, compression and shear tests are conducted on the micro model, which may be a unit cell (UC) or may contain multiple cells. The preferred micro model is called the representative volume element (RVE). Unit cell is used in this study. Somnic and Jo [6] presented a concise review of homogenization methods used for lattice materials.

Deshpande et al. [3] have published elastic modulus E_D and shear modulus G_D of octet-truss unit cell with Equations (2a-b) in terms of aspect ratio as follows.

$$\frac{E_D}{E_b} = \frac{2\pi\sqrt{2}}{3}\left(\frac{R}{L}\right)^2 \tag{2a}$$

$$\frac{G_D}{E_b} = \frac{\pi}{\sqrt{2}}\left(\frac{R}{L}\right)^2 \tag{2b}$$

Here, E, G are the elastic modulus, shear modulus, respectively. The subscript b designates the properties of bulk material. The Poisson's ratio of the octet-truss lattice material has been reported as constant $\nu_D = 1/3$ independent from the relative density in reference [4].

Micro model, at which the detailed geometry is built by brick solid finite elements, provides the most actual representation of the material. Macro model, which is the continuous medium with attained homogenized material constants, minimizes the time and effort spent on large-scale structures, but some failure mechanisms like the availability to observe buckling is lost. On the other hand, modelling with beam finite elements is easier in processing and significantly reduces the required computer capacity and solution time compared to the micro model. Beam elements also offer the advantage of directly plotting the resultants. In this study, the octet-truss unit cell simulated by using brick finite elements is denoted by solid model and the same simulated by beam finite elements is called as beam model, shortly. Researchers have extensively used beam elements in the simulation of lattice structures [6-10] and they reported that solid model provided better predictions than the beam model. Various modification approaches of the beam model have focused on the compression testing of lattice materials and emphasize the determination of the elastic modulus. Generally, in most cases, the effect of shear stiffness is not significant next to bending stiffness. However, for the cases such as, tall beams with short spans, thin walled cross sections, beams subjected to concentrated loads, the effect of shear stiffness on deflection becomes important as mentioned in reference [11]. Octet-truss lattice material is cubic symmetric and shear modulus is needed for the complete prediction of the homogenized material behaviour.

The aim of this study is to investigate the beam model with and without modification, considering not only the elastic modulus but the shear modulus as well. Average stress homogenization method is used in order to find the effective material constants of octet-truss lattice material for different values of relative density. Tensile and shear test simulations are performed on both the solid model and the beam models to have consistency with both sets of results.

2 Methods

The average stress method, identified among homogenization techniques in literature [12], is utilized to determine the effective material properties of composite materials. Determination of the composite material's stiffness matrix involves computing the necessary average stresses, as specified in the expression enclosed in the parentheses in Equation (3). However, for lattice materials featuring void spaces, adjustments are made to the average stress σ_{ij}^0 as follows.

$$\sigma_{ij}^0 = \left(\frac{1}{V_{st}}\int_V \sigma_{ij} dV\right)\frac{V_{st}}{V_{cell}} = \left(\frac{1}{V_{st}}\sum_{n=1}^{N}\sigma_{ij}^n V_n\right)\frac{V_{st}}{V_{cell}} \tag{3}$$

Here, N denotes the number of finite elements present within the model, V_n represents the volume of each finite element, V_{st} signifies the total volume encompassed by the struts, $V_{cell} = L_c^3$ is the volume of the cell including both the void space and the struts, σ_{ij}^n is the stress of each finite element. The stress and strain relation $\sigma_{ij} = C_{ij}\,\varepsilon_{ij}$ of the cubic symmetric material is defined by the stiffness matrix C_{ij}. Here, σ_{ij} and ε_{ij} denote the engineering stresses and strains of the unit cell, respectively. The cubic symmetry intrinsic to the octet-truss unit cell results in particular relationships among the components of the stiffness matrix: $C_{11} = C_{22} = C_{33}$, $C_{21} = C_{23} = C_{13}$ and $C_{44} = C_{55} = C_{66}$. The elastic behaviour of octet-truss lattice material can be described through three independent material constants: Elastic modulus E, Poisson's ratio ν and shear modulus G. Once the components of the stiffness matrix are obtained, these material constants can be computed as follows.

$$E_1 = E_2 = E_3 = (3\,C_{11}C_{21}^2 - C_{11}^3 - 2C_{21}^3)/(C_{21}^2 - C_{11}^2) \tag{4a}$$

$$\nu_{21} = \nu_{13} = \nu_{23} = (C_{21}^2 - C_{21}C_{11})/(C_{21}^2 - C_{11}^2) \tag{4b}$$

$$G_{21} = G_{13} = G_{23} = C_{44} \tag{4c}$$

Average stress homogenization method is applied on the solid model, beam model and the modified beam model. The results of solid model are presented without subscripts. The subscripts f, le and D are used to present the results obtained from the unmodified beam model, modified beam model and the results given by Deshpande et al. [3], respectively, as shown in Figure 1(b-d).

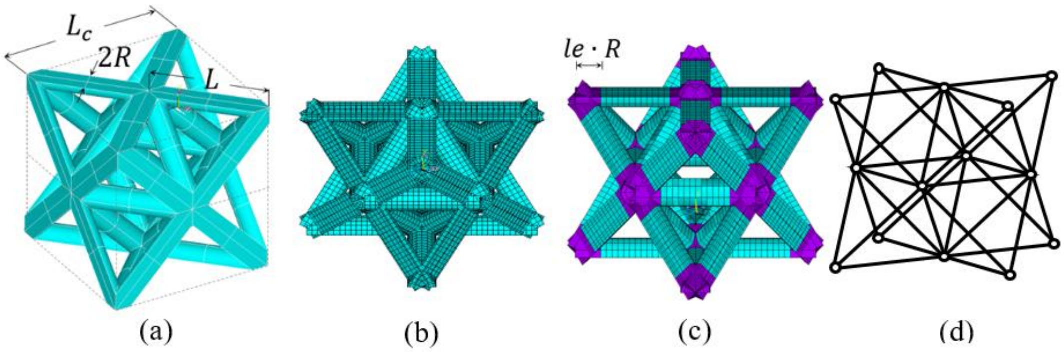

Figure 1: (a) Solid Model (E,G), (b) Beam Model (E_f, G_f), (c) Modified Beam Model (E_{le}, G_{le}), (d) Truss Model (E_D, G_D) [3].

Isotropic linear elastic bulk material is defined with Elastic modulus $E_b = 18\ GPa$ and Poisson's ratio $\nu_b = 0.3$, inspired by the study of Qi et al. [5], which was for Al-Si10-Mg. Here, the isotropic bulk material shear modulus can be calculated as $G_b = E_b/2/(1+\nu) = 6923.08MPa$. The unit cell dimensions used in the simulations are cell length $L_c = 6mm$ and strut length $L = 3\sqrt{2} = 4.24$mm, as shown in Figure 1(a). The simulations are repeated across the relative density range $\bar{\rho} = 0.01 - 0.5$ to capture the trend by including the required do loops within the text command files. Geometric data used in this study are tabulated in Table 1.

$\bar{\rho}$	$R\ (mm)$	R/L
0.01	0.08389	0.0197
0.10	0.27939	0.0658
0.20	0.41041	0.0967
0.30	0.52013	0.1226
0.40	0.62119	0.1464
0.50	0.71928	0.1695

Table 1: Octet truss aspect ratio R/L and relative density $\bar{\rho}$.

2.1 Homogenization of OT Unit Cell with Solid Model

Using the Parametric Design Language and Command references of Ansys APDL 2021 R1 [13] finite element program. For the solid model, the SOLID185 3-D 8-node structural brick finite element is used. The one-eighth of the solid model is shown in Figure 2(a) and the full solid model is shown in Figure 2(b).

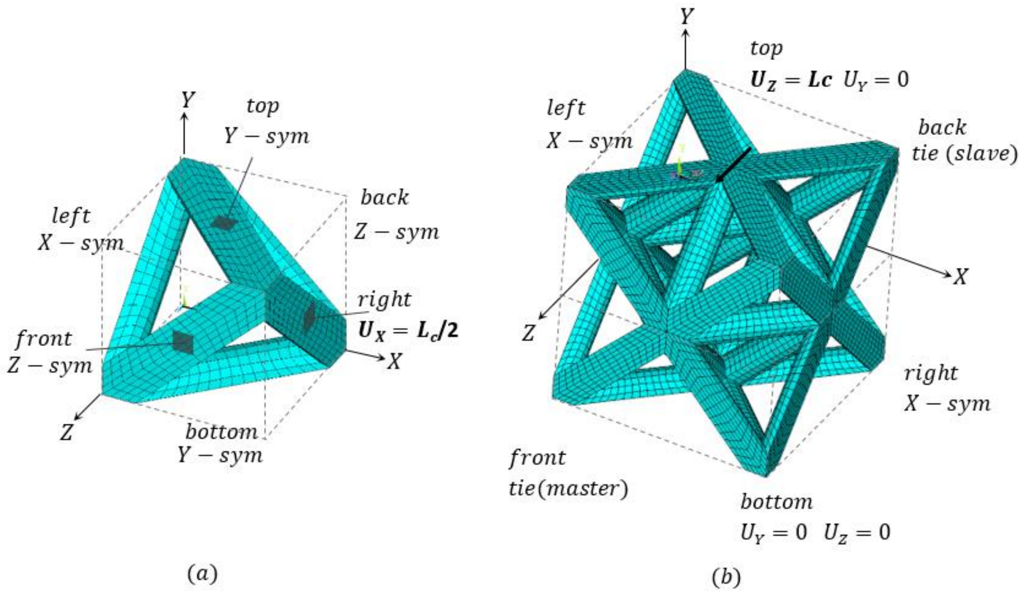

Figure 2: Boundary conditions for (a) one-eighth of unit cell for tensile test, (b) the entire unit cell for the shear test.

To derive the homogenized material properties, two tests are conducted on the unit cell. First, tensile test is performed using the boundary conditions on the one-eighth of the unit cell, as illustrated in Figure 2(a). The details of the boundary conditions are explained in reference [12]. These boundary conditions lead to the only non-zero strain of $\varepsilon_X = 1$. The average stress components are calculated using Equation (3) and they are directly equal to the first column components of the stiffness matrix C_{j1}. Then, the Elastic Modulus and Poisson ratio are calculated using Equations (4a-b). Second, the shear test simulation is conducted on the entire unit cell and the boundary conditions are explained in Figure 2(b). Symmetry boundary conditions are applied at the left and right surfaces as given in reference [12]. These boundary conditions lead to the only non-zero strain of $\gamma_{YZ} = 1$. The average stress component is calculated using Equation (3) and it is directly equal to the only nonzero component of the stiffness matrix C_{44} and the shear modulus as shown in Equation (4c).

2.2 Homogenization of OT Unit Cell with Beam Model

Beam model is constructed by beam elements perfectly connected at the joints. The struts of the octet-truss are simulated by BEAM188 3-D 2-node beam finite element available in Ansys APDL R1 [13]. Beam model is shown in Figure 1(b) with section display. Average stress method, that is explained for solid model, is applied also for the homogenization of beam models considered in this study.

2.3 Homogenization of OT Unit cell with Modified Beam Model

In this section, the same beam model is simulated but it is modified at the joints following the procedure found in literature [6-10] as shown in Figure 1(c). Modification of the beam model is based on two parameters. First one is the strut length of $le \cdot R$, on which the modification is applied, measured from the joint, as shown in Figure 1(c). The second parameter is ne, which is used to increase the stiffness over this strut length. The objective is to determine the modification parameters, le and ne, to be used in the beam model, so that the results of solid model can be obtained.

3 Results

3.1 Comparison of Solid Model and Beam Model

The results obtained from the solid model, beam model and Equations (2a-b) given by Deshpande et al. [3], are tabulated in Table 2 and plotted in Figure 3.

	Solid Model			Deshpande et al. [3]			Unmodified Beam Model		
$\bar{\rho}$	E/E_b	G/G_b	ν	E_D/E_b	G_D/G_b	ν_D	E_f/E_b	G_f/G_b	ν_f
0.01	0.0012	0.0023	0.33	0.0012	0.0023	0.33	0.0012	0.0023	0.33
0.10	0.0152	0.0279	0.33	0.0128	0.0251	0.33	0.0133	0.0256	0.33
0.20	0.0375	0.0655	0.33	0.0277	0.0541	0.33	0.0290	0.0565	0.32
0.30	0.0693	0.1145	0.32	0.0445	0.0868	0.33	0.0491	0.0928	0.32
0.40	0.1143	0.1779	0.30	0.0635	0.1238	0.33	0.0724	0.1355	0.31
0.50	0.1772	0.2597	0.29	0.0851	0.1660	0.33	0.1003	0.1860	0.30

Table 2: Octet truss material constants obtained by average stress method.

It is observed from Figure 3 that the solid model and beam model are in good agreement for low values of relative density, $\bar{\rho} < 0.1$, but for higher values, beam model underestimates the solid model results. Ushijima et al. [8] reported that the finite element results using beam elements and solid elements are in good agreement if the strut aspect ratio is relatively small ($d/L < 0.1$), where d is the strut diameter. This aspect ratio corresponds to $R/L < 0.05$ and $\bar{\rho} < 0.1$ for octet-truss lattice material, see Table 1. For higher values of relative density, the beam model suffers from the lack of material contact at the joints. The differences between the solid model and the unmodified beam model results can be calculated from Table 2. The elastic modulus values found with beam model for $\bar{\rho} = 0.1 - 0.5$ are $12\% - 43\%$ smaller than the ones found from solid model, respectively. The same is correct for the shear modulus values with the percentages of $8\% - 28\%$ as well.

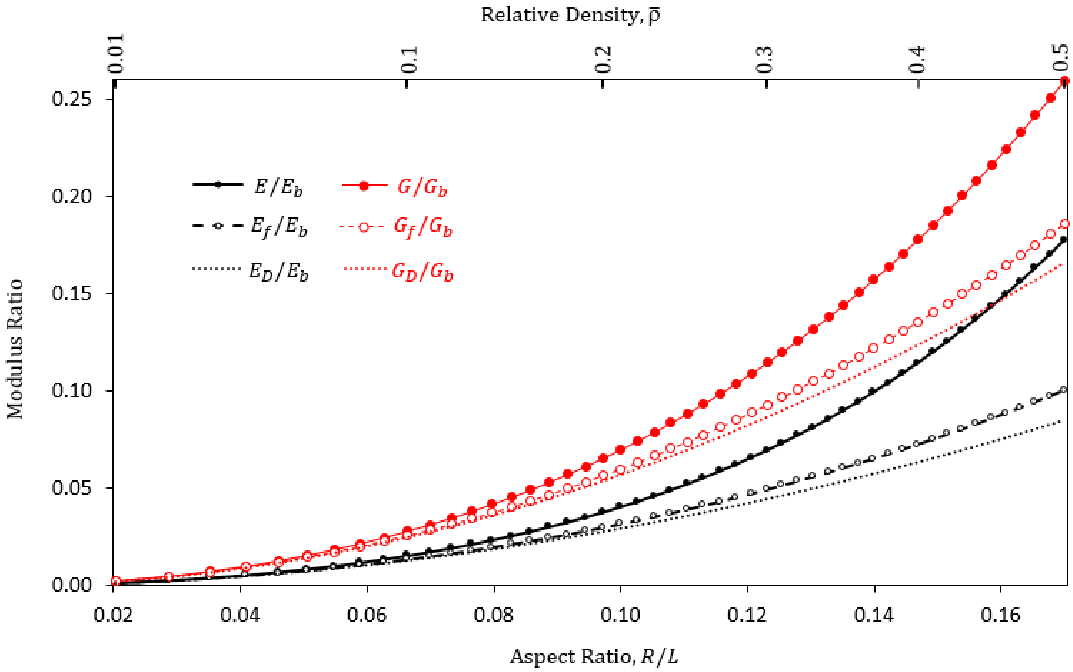

Figure 3: Elastic and shear modulus values of octet-truss unit cell, normalized by the bulk material constants E_b, G_b and obtained from solid model (E, G), unmodified beam model (E_f, G_f), Deshpande et al., [3] (E_D, G_D), respectively, with respect to the aspect ratio R/L and volume density $\bar{\rho}$.

Either the relative density $\bar{\rho}$ or the aspect ratio R/L can be used as a metric for characterization of the mechanical behaviour of the lattice materials as mentioned in reference [2]. If we adapt this idea for the material constants of octet-truss used in this study, the relations in Equations (5a-b) are obtained by means of curve fitting on the solid model results for $\rho = 0.1 - 0.5$ as follows.

$$\frac{E}{E_b} = 17.56 \left(\frac{R}{L}\right)^{2.61} \tag{5a}$$

$$\frac{G}{G_b} = 16.30 \left(\frac{R}{L}\right)^{2.35} \tag{5b}$$

Here, the coefficient of determination of Equation (5a) and (5b) are 0.99623 and 0.99828, respectively. And the extremum error is 6.3% with Equation (5a) and that of with Equation (5b) is 3.2% in the range of $\rho = 0.1 - 0.5$. It is observed that, if the unmodified beam model is used for $\bar{\rho} < 0.1$, and Equations (5a) and (5b) is used for $\bar{\rho} \geq 0.1$, the error is less than 10%.

3.2 Investigation of the Modified Beam Model found in literature

The procedure carried out here to determine the modification parameters, le and ne, to be used in the beam model, is explained in detail for $\bar{\rho} = 0.5$ so that the results of solid model can be obtained. The subscript le is used to denote the moduli obtained from the modified beam model. First, for a constant le value, the tensile test is repeated to observe the $E_{le}(ne)$ variation. This procedure is repeated for other le values, as shown in Figure 4.

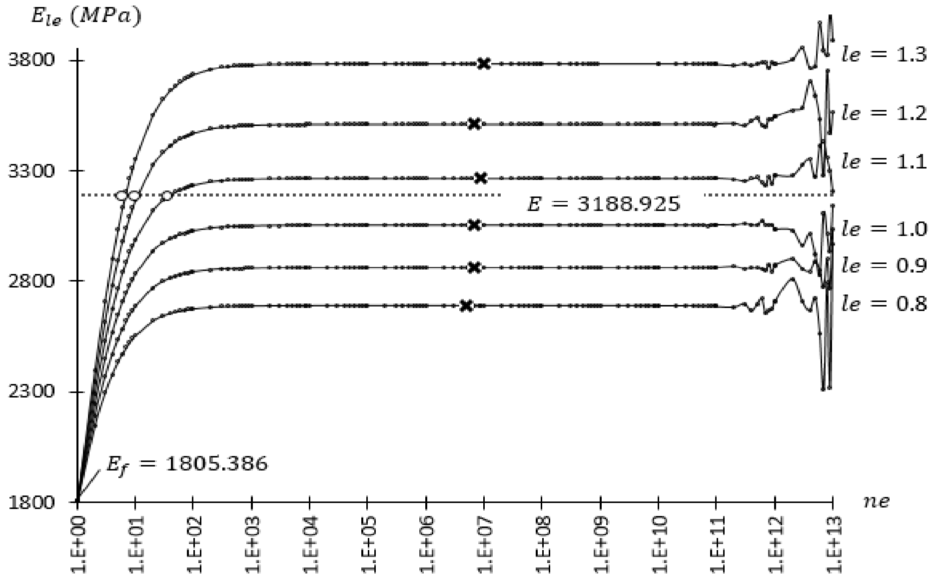

Figure 4: Modified beam model Elastic modulus variation with respect to ne, for constant le values ($\bar{\rho} = 0.5$).

For $ne = 0$, all the curves start at the same point, which is elastic modulus of the unmodified beam model, $E_f = 0.1003 \cdot 18000 = 1805.4\, MPa$, as presented in Table 2. All curves exhibit a consistent trend, where the elastic modulus increases continuously with ne until it approaches to an asymptotic value. A slight scattering of about $1 MPa$ in magnitude starts around $ne = 10^6 - 10^7$, as indicated by the symbol × in Figure 4. Subsequently, a substantial scattering takes place at about $ne = 10^{11}$ due to numerical instability. The dotted horizontal line corresponds to solid model's elastic modulus value $E = 3188.925\, MPa$ for $\bar{\rho} = 0.5$. Thus, the intersection of this horizontal line with any curve gives the ne, le parameter pairs, to be used for the modification of the beam model to match the elastic modulus value of the solid model. It is noteworthy that only the curves for $le > 1.0654$ intersect with the horizontal solid model target value. For example, the intersection points for $le = 1.1$, 1.2 and

1.3 are marked by circles in Figure 4. The $le - ne$ curve for $\bar{\rho} = 0.5$ is plotted in Figure 5.

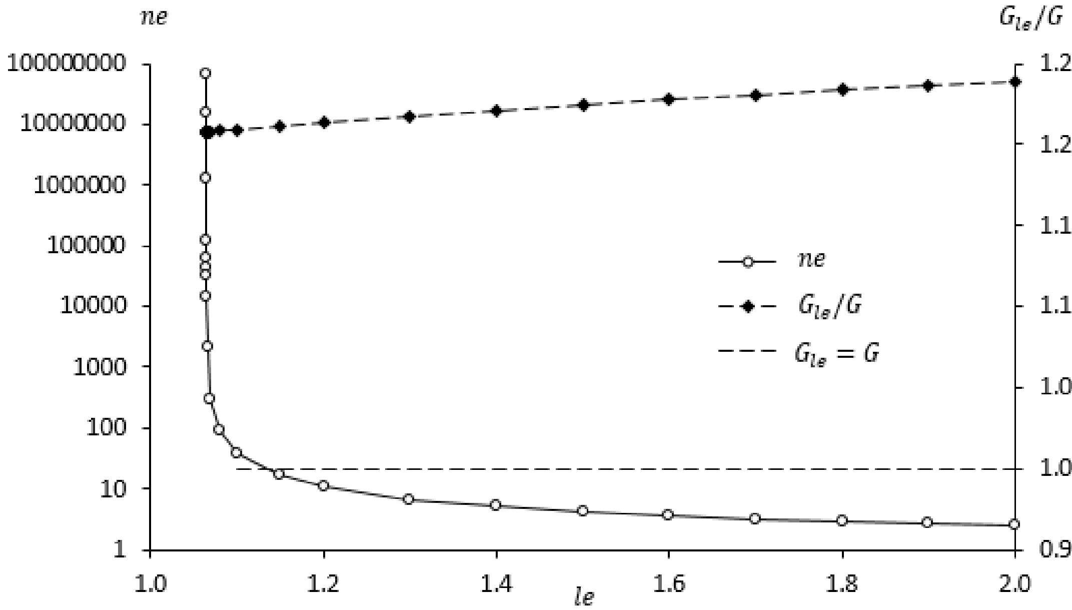

Figure 5: The graph showing the parameter pairs ne, le used in modified beam model to match to the solid model elastic modulus (-o-) on the primary axis and the calculated shear modulus on the secondary axis for octet-truss of $\bar{\rho} = 0.5$.

There are infinite number of parameter pairs those can be used to match the elastic modulus of the modified beam model with that of the solid model. Using these ne, le parameter pairs, shear modulus values G_{le} are calculated by running the shear test simulation. These values are normalized with the solid model shear modulus G and presented on the secondary vertical axis on the right side in Figure 5. The horizontal dashed line is included to signify the desired unit value, indicating a successful match between the shear modulus of beam model and the solid model. It can be observed from Figure 5 that, although there are infinite number of ne, le parameter pairs capable of aligning the elastic modulus values of the solid model and the modified beam model, but the shear modulus of modified beam model is always larger than that of solid model. And it does not help how much ne value is increased, the shear modulus of modified beam model fails to reach to the desired solid model shear modulus value. For a constant $ne = 10^4$ value, the shear modulus value obtained from the modified beam model for $\bar{\rho} = 0.5$ is at least 20.79% higher than the solid model's shear modulus value with this modification method found in literature.

4 Conclusions

Homogenized material constants of octet-truss cell are calculated using average stress method and the results are presented with respect to both the strut aspect ratio and the relative density $\bar{\rho} = 0.01 - 0.5$. Required tensile and shear test simulation are conducted with Ansys [13] finite element program.

This method is applied to both the micro solid model and the unmodified beam model, showing good agreement for small relative density values ($\bar{\rho} < 0.1$) but revealing underestimation by the beam model at higher relative densities. The elastic and shear modulus values obtained from the beam model are consistently smaller, with differences of 12%-43% and 8%-28%, respectively, compared to the micro solid model across $\bar{\rho} = 0.1 - 0.5$.

In literature, beam model modification usually enhances strut stiffness at joints, focusing on lattice material compression testing and emphasizing elastic modulus without considering shear modulus. This study incorporates both elastic and shear modulus in the modification, but it concludes that this approach fails to align both modulus values with solid model results.

Closed expressions for the homogenized moduli of octet-truss lattice material are presented in terms of aspect ratio with Equations (5a-b), by curve fitting on the results of the solid model for $\bar{\rho} = 0.1 - 0.5$. It is observed that, unmodified beam model can be used for $\bar{\rho} < 0.1$ and Equations (5a) and (5b) can be used for $\bar{\rho} \geq 0.1$ to keep the error under 10%.

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References

[1] C. Pan, Y. Han, J. Lu, "Design and optimization of lattice structures: A review", Applied Sciences, 10, 6374, 1-36, 2020.

[2] L. Gibson, M. Ashby, "Cellular Solids: Structure and Properties", 2nd ed, Cambridge University Press, 1997.

[3] V.S. Deshpande, N.A. Fleck, M.F. Ashby, "Effective properties of the octet-truss lattice material", Journal of the Mechanics and Physics of Solids, 49, 1747-1769, 2001, doi: 10.1016/S0022-5096(01)00010-2

[4] T.T. Dejean, A.B. Spierings, D. Mohr, "Additively-manufactured metallic micro-lattice materials for high specific energy absorption under static and dynamic loading", Acta Materialia, 116, 14-28, 2016.

[5] D. Qi, H. Yu, M. Liu, H. Huang, S. Xu, Y. Xia, G. Qian, W. Wu, "Mechanical behaviors of SLM additive manufactured octet-truss and truncated-octahedron lattice structures with uniform and taper beams", International Journal of Mechanical Sciences, 163, 105091, 2019.

[6] J. Somnic, B.W. Jo, "Review: Status and challenges in homogenization methods for lattice materials", Materials, MDPI, 15, 605, 2022.

[7] M.H. Luxner, J. Stampfl, H.E. Pettermann, "Finite element modeling concepts and linear analysis of 3D regular open cell structures", Journal of Materials Science, 40, 22, 5859-5866, 2005.

[8] K. Ushijima, W.J. Cantwell, R.A.W Mines, S. Tsopanos, M. Smith, "An investigation into the compressive properties of stainless-steel micro-lattice structures", Journal of Sandwich Structures and Materials, 13, 3, 303-329, 2010.

[9] M. Smith, Z. Guan, W.J. Cantwell, "Finite element modelling of the compressive response of lattice structures manufactured using the selective laser melting technique", International Journal of Mechanical Sciences, 67, 28-41, 2013.

[10] M. Zhao, X. Li, D.Z. Zhang, W. Zhai, "Geometry effect on mechanical properties and elastic isotropy optimization of bamboo-inspired lattice structures", Additive Manufacturing, 64, 103438, 2023.

[11] M. Inan, "Strength of Materials", ITU Vakfi Yayinlari, translated by S. Sami, Istanbul, Turkey, ISBN: 9786059581158, 2019.

[12] E.J. Barbero, "Finite Element Analysis of Composite Materials using Abaqus", CRC Press, New York, 2013.

[13] Ansys Release 2021 R1, Mechanical APDL Contact Technology Guide, 2021.