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Resistance Model Uncertainty in Non-Linear Numerical Analyses of Ultra-High-Performance Reinforced Concrete Beams in Flexure

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Abstract

This study presents the bending resistance model uncertainty and corresponding partial factors when performing a design or an assessment of ultra-high-performance reinforced concrete (UHPC) beams via non-linear finite element analyses (NLFEA). UHPC beams that have been both experimentally tested and simulated via NLFEA are considered, as documented in the literature, treating each source as presenting a unique modelling hypothesis of the beams' bending behaviour. A probabilistic analysis through Bayesian updating processes these uncertainties, updating prior distributions of resistance model uncertainty with data from various modelling hypothesis to estimate posterior distributions and the final average posterior distribution. The coefficient of variation and mean value of the average posterior distribution is used to calibrate corresponding partial factors in accordance with the the global safety format for NLFEAs proposed by codes and literature.

Keywords: ultra-high-performance concrete, non-linear finite element analysis, flexure, resistance model uncertainties, partial factor, Bayesian updating.

1 Introduction

Optimizing the performance of ultra-high-performance concrete (UHPC) beams under bending stresses is critical for advancing resilient construction technologies [1,2]. Traditional analytical models often fail to accurately predict the non-linear responses of UHPC beams, leading to significant uncertainties in structural behavior assessments [3–5]. This underscores the importance of non-linear finite element analysis (NLFEA) in the design and assessment of these structures, as it can provide detailed insights into the complex interactions under flexural loading, crucial for ensuring safety and durability [6] and reducing overly crude estimates provided by simplified analytical models [7]. Recent advancements in NLFEA have improved our understanding of UHPC beam behaviors, particularly in bending scenarios. Research by Yoo and Yoon [8], Shafieifar et al. [9], Liu et al. [10], Zhu et al. [11], Simwanda et al. [12] and Zhu et al. [13] has demonstrated the NLFEA is effective in simulating critical stress points and deformation patterns, influencing design optimizations that meet the rigorous demands of modern infrastructures. However, despite these advancements, the application of NLFEA still faces challenges due to model uncertainties and the variability in material and geometric properties that are not fully accounted for in existing models [3,4]. This paper addresses these gaps by applying a methodology set forth by previous studies of Castaldo et al. [14,15] to quantify resistance model uncertainties and partial factors in UHPC beams subjected to pure bending moments. The contribution aims to enhance the design and evaluation of UHPC structures via non-linear NLFEAs, promoting the development of beams that meet the target reliability levels set in design standards.

2 Methodology for quantifying uncertainty in NLFEAs

In the assessment of new and existing of UHPC beams in bending vial partial factor or full probabilistic methods, understanding and quantifying epistemic (or model) uncertainty in the resistance model definition is paramount [16,17]. This section explores a framework for addressing such uncertainties, which stem from limitations in knowledge about the structure's behaviour, and can significantly affect the outcomes of NLFEAs as depicted in Figure 1. Unlike aleatory uncertainties, which are associated with inherent randomness of the system, epistemic uncertainties can be reduced with enhanced understanding and more comprehensive data [18]. Central to this evaluation is the calculation of the global design structural resistance, R_d, through the formula [19]:

$$R_d = \frac{R_{rep}}{\gamma_R \gamma_{Rd}} \tag{1}$$

where R_{rep} is the representative value of global bending resistance derived from NLFEAs, γ_R encompasses the aleatory uncertainties related to material properties, and γ_{Rd} s the

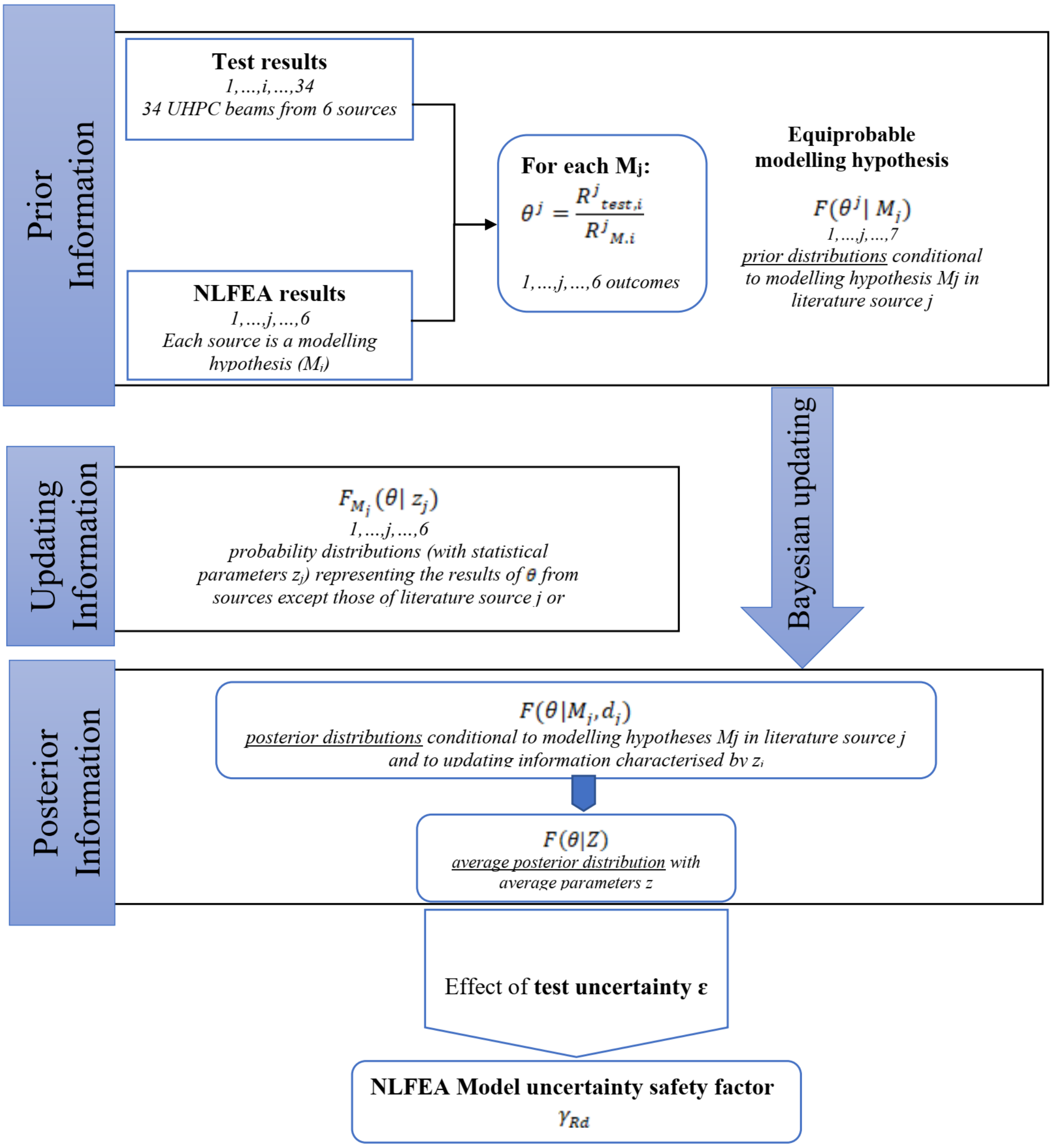

Figure 1: Framework for calibration of NLFEA model uncertainty safety factor γ_{Rd} in accordance with the global safety factor format.

resistance model uncertainty factor addressing epistemic uncertainties within the definition of the non-linear structural model simulating a bending test [19].

Given the complex nature of modelling UHPC beams [18], and the lack of established analytical models [3,9], it's crucial to accurately estimate the partial factor γ_{Rd}. The assessment procedure adopts a comparative approach to compute the NLFEA resistance model uncertainty via the ratio [16, 17].

$$\theta = \frac{R(X, Y)}{R_{NLFEA}(X)} \tag{2}$$

which contrasts the global bending resistance obtained from experimental investigations $R(X, Y)$ with that derived from NLFEA $R_{NLFEA}(X)$. X denotes the vector of basic variables - inputs into the model and Y is the vector of variables that may affect resistance but are not explicitly considered in the model. This comparison aids in characterising the random variable that encapsulates the resistance model uncertainty in the NLFEA bending capacity test. To quantify γ_{Rd} accurately, Bayesian updating is employed to refine the probabilistic model representing the resistance model uncertainty [14]. This process integrates prior distribution, based on initial experimental and numerical data, with new evidence from other experimental and numerical studies to update the probabilistic model. The updated model then facilitates the calculation of γ_{Rd} as follows [16, 17]:

$$\gamma_{Rd} = \frac{1}{\mu_\theta} \exp(\alpha_R \beta V_\theta) \tag{3}$$

where:

- μ_θ, the mean of the NLFEA resistance model uncertainty,
- β, the target reliability index reflecting the desired level of reliability,
- α_R, the first order reliability method (FORM) sensitivity factor, and
- V_θ, the coefficient of variation of the NLFEA resistance model uncertainty.

3 NLFEA model uncertainty across literature sources

This analysis considers experimental tests on the bending capacity behaviour of UHPC beams, as conducted by the following studies: [3, 8–11, 13] (referred to as literature sources 1 through 6 hereafter), alongside counterpart NLFEA for these specimens. Instead of delving deeply into the varied assumptions and hypotheses of each literature source, this study simplifies the approach by treating each source as a distinct modelling strategy. Specifically, six different modelling hypotheses, from M_1 to M_6, are evaluated. Tables 1 to 6 display a comparison between the numerically computed and experimentally determined global peak bending capacities. Each source's modeling hypothesis is quantified through the prior parameters, mean (μ) and standard deviation (σ), of the model uncertainty, as shown in the respective tables.

The assessment of modelling uncertainty in various sources has shown that NLFEA models provide consistent but variably precise predictions for the bending capacities of UHPC beams. Variations in θ values, representing the ratios of test to NLFEA results, highlight potential discrepancies due to different modelling assumptions or experimental setups, with some sources indicating possible inaccuracies. Additionally, the small standard deviations across hypotheses generally underscore the models' stability and reliability. Nevertheless, slight variations in the mean values of θ suggest differing degrees of conservatism, likely influenced by beam geometries and material specifics.

Ref.	Specimen	$R_{test,i}$ (kNm)	$R_{NLFEA,i}$ (kNm)	θ	Modelling hypothesis
[8]	NF-0.94	28.2	27.1	1.04	M$_1$
	NF-1.50	44.1	40.5	1.09	
	S13-0.94	39.3	37.1	1.06	
	S13-1.50	55.8	50.5	1.10	$(\mu_\theta = 1.08, \sigma_\theta = 0.03)$
	S19.5-0.94	42.0	39.5	1.06	
	S19.5-1.50	56.3	52.1	1.08	
	S30-0.94	43.2	38.8	1.11	

Table 1: NLFEA model uncertainty and peak resistances from both experimental tests and NLFEA from literature source 1.

Ref.	Specimen	$R_{test,i}$ (kNm)	$R_{NLFEA,i}$ (kNm)	θ	Modelling hypothesis
[9]	S6x6-2.6-0.85	54.3	50.6	1.07	M$_2$
	S6x6-1.7-0.65	27.2	30.535	0.89	
	S4x6-3.9-0.75	30.4	28.655	1.06	$(\mu_\theta = 1.05, \sigma_\theta = 0.15)$
	S2x6-1.8-0.50	3.3	2.565	1.29	
	S6x6-0.6-0.85	23.2	24.85	0.93	

Table 2: NLFEA model uncertainty and peak resistances from both experimental tests and NLFEA from literature source 2.

Ref.	Specimen	$R_{test,i}$ (kNm)	$R_{NLFEA,i}$ (kNm)	θ	Modelling hypothesis
[10]	T-1	173	190	0.91	M$_3$
	T-2	236.5	267.5	0.88	
	T-3	286.5	310	0.92	$(\mu_\theta = 0.96, \sigma_\theta = 0.08)$
	T-4	297.3	322.5	0.92	
	T-5	281.6	300	0.94	

Table 3: NLFEA model uncertainty and peak resistances from both experimental tests and NLFEA from literature source 3.

Ref.	Specimen	$R_{test,i}$ (kNm)	$R_{NLFEA,i}$ (kNm)	θ	Modelling hypothesis
[11]	B1-2	180	170	1.06	M$_4$
	B2-2	217	198	1.10	
	B2-3	227.5	205	1.11	$(\mu_\theta = 1.03, \sigma_\theta = 0.08)$
	B3-2	271.5	246	1.10	

Table 4: NLFEA model uncertainty and peak resistances from both experimental tests and NLFEA from literature source 4.

Ref.	Specimen	$R_{test,i}$ (kNm)	$R_{NLFEA,i}$ (kNm)	θ	Modelling hypothesis
[13]	A-PBE	211.1	224.4	0.94	M$_5$
	A-PB	174	184.6	0.94	
	C-BE	138.35	142.9	0.97	$(\mu_\theta = 1.02, \sigma_\theta = 0.08)$
	C-B	115.7	119.171	0.97	

Table 5: NLFEA model uncertainty and peak resistances from both experimental tests and NLFEA from literature source 5.

4 Bayesian updating

4.1 Prior Models and Updating Information

This section details the statistical information related to the prior distribution of model uncertainty and the corresponding updating information used in the Bayesian updat-

Ref.	Specimen	$R_{test,i}$ (kNm)	$R_{NLFEA,i}$ (kNm)	θ	Modelling hypothesis
[3]	RSC1	45	41	1.10	M_6
	RSC2	46.5	42.5	1.09	
	RSC4	48.5	43	1.13	
	S4x6-3.9-0.75	125	112	1.12	
	S6x6-1.7-0.85	175	160	1.09	$(\mu_\theta = 1.09, \sigma_\theta = 0.04)$
	S6x6-2.6-0.85	225	205	1.10	
	R12-2	74.5	73.25	1.02	
	R13-2	95	85.60	1.11	
	R14-2	105	96	1.09	

Table 6: NLFEA model uncertainty and peak resistances from both experimental tests and NLFEA from literature source 6.

ing process to derive the posterior distribution. The distributions for the prior models and the updating data were all assumed to be lognormally distributed [14]. As illustrated in Fig. 1, the prior model for each modeling hypothesis i is derived from model uncertainty computations based on literature source i. Conversely, the updating information for M_i encompasses all model uncertainty data from sources other than source i. These statistical properties are represented in terms of probability density functions (PDFs) and cumulative distribution functions (CDFs), as shown in Fig. 2.

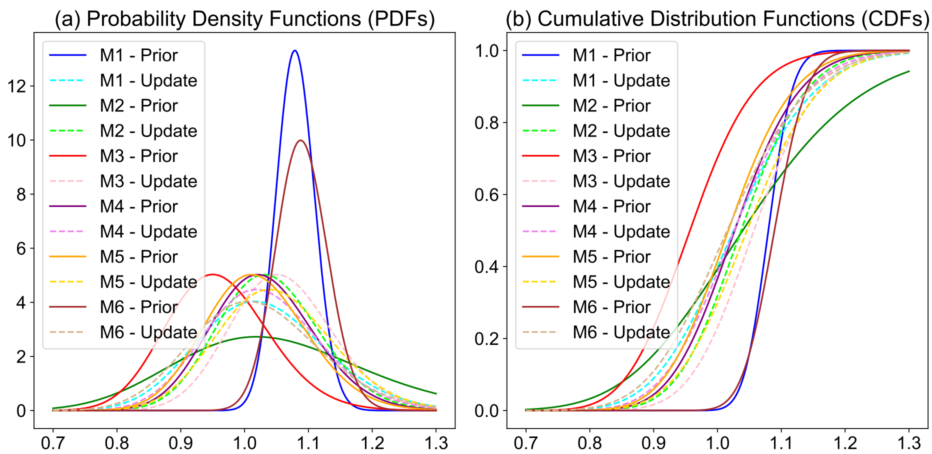

Figure 2: Statistical information of prior models and updating information represented in terms of (a) PDFs, and (b) CDFs

The PDFs and CDFs for various models (M1 through M6) in Fig. 3 exhibit significant differences in peak heights and spreads, indicating variable levels of variability and bias estimates. Notably, the prior models for M1 and M6 tend to show lower variability but higher bias compared to other models. Excluding M1 and M6, the prior models for other modeling hypotheses display similar statistical properties compared to the distributions of the updating information ('updates'), suggesting that modeling hypotheses or experimental setup errors in M1 and M6 may contribute to the observed differences in bias and variability. To mitigate the effects of modeling or experimental errors on the final model uncertainty representation, a Bayesian updating approach is employed. This method robustly processes the uncertainties by incorporating information from both the prior models and updating information, thereby refining the posterior distribution.

4.2 Posterior distributions

The Bayesian updating process, as visualized in the flowchart (Fig. 1), enhances our understanding of model uncertainty through a probabilistic framework. For each set of UHPC beam test results and corresponding NLFEA outcomes, an individual posterior distribution is formulated. These distributions represent an advanced level of knowledge, refining the prior distribution parameters based on the updating information collected across a wide range of sources. Incorporating 34 UHPC beam results from 6 sources and the respective NLFEA results for 6 distinct modelling hypotheses, this analysis converges to a singular posterior distribution. As depicted in Table 7, the statistical parameters of prior, updating, and posterior distributions follow the lognormal assumption, with the mean values converging towards unity and standard deviations reducing, indicating an increased confidence in the model post updating.

Modelling	Prior distribution		Updating information		Posterior distribution	
hypothesis	μ_θ $[-]$	σ_θ $[-]$	μ_θ $[-]$	σ_θ $[-]$	μ_θ $[-]$	σ_θ $[-]$
M1	1.08	0.03	1.03	0.1	1.00	0.052
M2	1.05	0.15	1.04	0.08	1.00	0.050
M3	0.96	0.08	1.06	0.08	1.00	0.046
M4	1.03	0.08	1.03	0.09	1.00	0.045
M5	1.02	0.08	1.05	0.09	1.00	0.045
M6	1.09	0.04	1.02	0.1	1.10	0.033
Average					1.02	0.045

Table 7: Statistical parameters of prior, posterior, and updating information for Lognormal distribution functions.

The posterior distributions signify a comprehensive characterisation of the model uncertainty and serve as the probabilistic foundation for evaluating the resistance model uncertainty safety factor γ_{Rd}. This factor accounts for the combined effect of test uncertainty and the inherent variance within the NLFEA results, ensuring a robust, reliability-based approach to structural analysis and design.

Important to note is that observed model characteristics (nearly unity mean and CoV of about 5%) are favourable. It is expected that for larger database covering wider ranges of material properties and geometries, particularly variability would increase. However, it seems to be evident that NLFEA models offer significant improvements in comparison to simplified analytical models [7] for which CoV of model uncertainty often exceeds 30%, becoming a major source of uncertainty for highly engineered UHPC structural members.

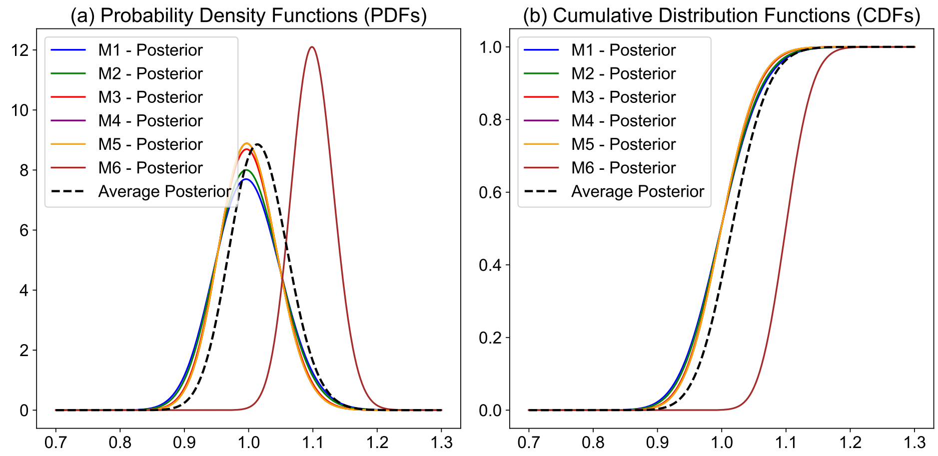

Figure 3: Statistical information of posterior distributions and the average posterior distribution represented in terms of (a) PDFs, and (b) CDFs

5 Partial factors

In the context of resistance model uncertainty in NLFEA of UHPC beams in flexure, the calibration of partial safety factors γ_{Rd} is crucial. These factors are determined by considering the target reliability indices β prescribed by design codes and literature for both existing and new structures [14, 17]. These reliability indices account for varying consequences of structural failure, which are intrinsically tied to human safety and the anticipated service life of the structure [17, 18]. For the non-dominant resistance variable case, where the resistance model uncertainty is considered less influential compared to material and geometric uncertainties Table 8 outline the calibrated partial factors γ_{Rd} under different conditions for service life, consequences of failure, and target reliability indices β, using a FORM sensitivity factor α_R.

New structures	Service life [years]	Consequences of failure	Reliability index β [-]	FORM factor α_R [-]	Partial factor γ_{Rd} [-]
	50	Low	3.1	0.32	1.05
	50	Medium	3.8	0.32	1.06
	50	High	4.4	0.32	1.07

Existing structures	Reduced service life [years]	Reliability index β [-]	FORM factor α_R [-]	Partial factor γ_{Rd} [-]
	50	3.1-3.8	0.32	1.05-1.06
	15	3.4-4.1	0.32	1.05-1.06
	1	4.1-4.7	0.32	1.06-1.07

Table 8: Partial factors γ_{Rd} at different target reliability levels for NLFEAs of UHPC beams in flexure considering that the resistance variable is non-dominant

6 Conclusions and contributions

This study has investigated the resistance model uncertainties in NLFEA of UHPC beams subjected to bending loads, using Bayesian updating to refine understanding and quantification of these uncertainties. It was found that in contrast simplified analytical models, NLFEA models effectively predict the bending resistance of UHPC beams, despite the inherent complexities of modeling high-performance materials. The precision varies across different sources, highlighted by statistical analysis. Bayesian updating significantly improved the accuracy of estimated partial factors (γ_{Rd}), better aligning them with the actual performance observed in experimental setups. The submitted study based on the limited database indicates that model uncertainty for NLFEA of UHPC beams in flexure indicates might be characterised by a unity mean and coefficient of variation of 5%, with a corresponding partial factor of 1.06 (model uncertainty factor). However, further investigations focused on wider ranges of material properties and beams' geometries are required to provide recommendations for reliability analyses of UHPC beams.

The methodological approach developed herein for quantifying and updating resistance model uncertainty can be generalized to other structural analysis contexts, improving the reliability and safety of engineered structures. This study introduces a comprehensive framework for updating model uncertainties in NLFEA of UHPC beams, refines the methodology for calibrating partial factors based on probabilistic models, and extends the application of Bayesian statistical methods in civil engineering. These contributions not only pave the way for more reliable structural designs but also enhance the understanding of UHPC behavior under load, promoting safer and more efficient construction practices.

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References

[1] I. H. Yang, C. Joh, B. S. Kim, "Structural behavior of ultra high performance concrete beams subjected to bending," Eng. Struct., 2010. DOI:10.1016/j.engstruct.2010.07.017.

[2] L. Simwanda, N. De Koker, C. Viljoen, A. J. Babafemi, "Structural reliability of existing rc beams strengthened with uhpfrc tensile lay-

ers," Acta Polytechnica CTU Proceedings, vol. 36, pp. 191-197, 2022. DOI:10.14311/APP.2022.36.0191.

[3] L. Simwanda, N. De Koker, C. Viljoen, "Structural reliability of ultra high-performance fibre reinforced concrete beams in flexure," Engineering Structures, vol. 244, 112767, 2021. DOI:https://doi.org/10.1016/j.engstruct.2021.112767.

[4] L. Simwanda, A. J. Babafemi, N. D. Koker, C. Viljoen, "Bayesian calibration and reliability analysis of ultra high-performance fibre reinforced concrete beams exposed to fire," Structural Safety, vol. 103, 102352, 2023. DOI:https://doi.org/10.1016/j.strusafe.2023.102352.

[5] L. Simwanda, N. De Koker, C. Viljoen, A. J. Babafemi, "Structural reliability of ultra-high-performance fiber reinforced concrete beams in shear," Structural Concrete, vol. 24, pp. 2862-2878, 2023. DOI:10.1002/suco.202200342.

[6] M. Qiu, X. Shao, Y. Zhu, J. Zhan, B. Yan, Y. Wang, "Experimental investigation on flexural cracking behavior of ultrahigh performance concrete beams," Struct. Concr., vol. 21, pp. 2134-2153, 2020. DOI:10.1002/suco.201900339.

[7] D. Y. Yoo, Y. S. Yoon, "Structural performance of ultra-high-performance concrete beams with different steel fibers," Engineering Structures, 2015. DOI:10.1016/j.engstruct.2015.08.029.

[8] M. Shafieifar, M. Farzad, A. Azizinamini, "A comparison of existing analytical methods to predict the flexural capacity of ultra high performance concrete (UHPC) beams," Construction and Building Materials, 2018. DOI:10.1016/j.conbuildmat.2018.03.229.

[9] C. Liu, Y. Zhang, Y. Yao, Y. Huang, "Calculation method for flexural capacity of high strain-hardening ultra-high performance concrete t-beams," Structural Concrete, vol. 20, pp. 405-419, 2019. DOI:10.1002/suco.201800151.

[10] L. Zhu, T. Li, Y. Wang, Q. Tang, G. Wang, Z. Zhang, "Experimental and numerical study on t-shaped uhpfrc beams with high-strength reinforcement," Structural Concrete, vol. 22, pp. 3630-3645, 2021. DOI:10.1002/suco.202100067.

[11] L. Simwanda, C. Kahanji, F. Ali, "Numerical modelling, parametric analysis and design of uhpfrc beams exposed to fire," Structures, vol. 52, pp. 1-16, 2023. DOI .org/10.1016/j.istruc.2023.03.155.

[12] Y. Zhu, Y. Zhang, J. Shi, "Finite element analysis of flexural behavior of precast segmental uhpc beams with prestressed bolted hybrid joints," Engineering Structures, vol. 238, 111492, 2021. DOI:https://doi.org/10.1016/j.engstruct.2020.111492.

[13] P. Castaldo, D. Gino, G. Bertagnoli, G. Mancini, “Partial safety factor for resistance model uncertainties in 2d non-linear finite element analysis of reinforced concrete structures,” Engineering Structures, vol. 176, pp. 746-762, 2018. DOI .org/10.1016/j.engstruct.2018.09.041.

[14] P. Castaldo, D. Gino, G. Bertagnoli, G. Mancini, “Resistance model uncertainty in non-linear finite element analyses of cyclically loaded reinforced concrete systems,” Engineering Structures, vol. 211, 110496, 2020. DOI .org/10.1016/j.engstruct.2020.110496.

[15] M. Holický, J. V. Retief, M. Sýkora, “Assessment of model uncertainties for structural resistance,” Probabilistic Engineering Mechanics, vol. 45, pp. 188-197, 2016. DOI:10.1016/j.probengmech.2015.09.008.

[16] International Federation for Structural Concrete (fib), “fib Model Code for Concrete Structures 2020 (final draft),” Case Postale 88, CH-1015 Lausanne, Switzerland, 2023.

[17] A. D. Kiureghian, O. Ditlevsen, “Aleatory or epistemic? does it matter?,” Structural Safety, vol. 31, pp. 105-112, 2009. DOI .org/10.1016/j.strusafe.2008.06.020.

[18] D. L. Allaix, V. I. Carbone, G. Mancini, “Global safety format for non-linear analysis of reinforced concrete structures,” Structural Concrete, vol. 14, pp. 29-42, 2013. DOI:10.1002/suco.201200017.

[19] M. Sykora, W. Botte, R. Caspeele, D. Diamantidis, A. Muttoni, R.D.J.M. Steenbergen, ”Probabilistic Models for Resistance Variables in fib Model Code 2020 for Design and Assessment,” in *Proceedings of International Probabilistic Workshop IPW 2022*, pp. 206-215, Stellenbosch, South Africa, September 8-9, 2022. Czech Technical University in Prague, ISSN 2336-5382.