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Development of Strain-Based Approach for Safety Assessment of RC Systems using Non-Linear Numerical Methods

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Abstract

Refined non-linear numerical analyses can be a powerful tool for evaluating the safety level of both new and existing RC structural systems accurately. Researchers and code-makers have made significant efforts to define suitable safety formats to meet reliability requirements using refined non-linear numerical analyses. However, employing refined non-linear numerical analyses can be time-consuming with practical challenges. This study describes the basis for a method that minimizes the need for refined non-linear numerical analyses while providing an accurate estimate of structural resistance. The statistical parameters characterizing the probabilistic distribution of global structural resistance can be determined by fitting equations based on extensive probabilistic investigations, considering the peak strain in the main reinforcement. This strain can be estimated through a refined non-linear numerical analysis of the RC member using mean material properties and nominal geometrical ones. The estimation of these statistical parameters facilitates the assessment of partial safety factors within a semi-probabilistic framework for practical applications.

Keywords: non-linear numerical analysis, global safety format, structural safety, strain-based method, RC structures, finite elements method.

1 Introduction

Considerable research efforts have been dedicated to develop methodologies facilitating practical applications of Non-linear Numerical Analyses (NLNAs) for assessing reinforced concrete (RC) structures [1]-[8]. This trend has emerged over recent decades, driven by advancements in computational capabilities and the adoption of risk-based approaches for safety, societal and economic evaluations [9]-[15]. There is a growing expectation that NLNAs will become increasingly adopted by practitioners worldwide for addressing complex structural problems [16]. However, integrating NLNAs into safety assessments presents computational challenges, necessitating specialized skills from analysts [17]-[18]. While NLNAs offer significant insights into structural behaviour, their complexity can represent a challenge for designers [19]-[20]. Nonetheless, widespread acceptance and incorporation of NLNAs into the next generation of design codes are anticipated, as evidenced in prEN 1992-1-1:2021 [21]. In this context, the efficacy of mathematical approaches in accurately predicting structural responses must align with safety requirements outlined in current international codes such as *fib* Model Code 2010 [22], *fib* Model Code 2020 [23] and EN1992-1-1:2014 [24].

In the last ten years, academic literature has presented several safety formats for NLNAs of RC elements or systems [25]-[26], predominantly relying on the "global resistance method" (GRM) (*fib* Model Code 2010 [22]). The GRM is used to determine the design value of structural resistance (R_d), as depicted in Eq. (1):

$$R_d = \frac{R_{NLNA}(f_m; a_n)}{\gamma_R \cdot \gamma_{Rd}} \tag{1}$$

where $R_{NLNA}(f_m; a_n)$ represents the global structural resistance assessed using mean material properties (f_m) and nominal geometrical values (a_n); γ_R is the partial safety factor accounting for the variability in material and geometrical properties (i.e., the aleatory component of overall uncertainties) and γ_{Rd} represents the partial safety factor associated with modeling uncertainties (i.e., the epistemic component of overall uncertainties). Both partial safety factors can be defined as function of a specific target reliability level [3],[22]. Literature [17]-[18] offers pertinent recommendations for the direct application and estimation of γ_{Rd}. Conversely, determining γ_R depends on the safety format [4]. Generally, if the global structural resistance follows a lognormal distribution (*fib* Model Code 2010 [22]), the value of γ_R can be estimated as follows:

$$\gamma_R = \frac{\exp\left(\alpha_R \beta_T \cdot V_R\right)}{\delta_R} \tag{2}$$

where V_R represents the coefficient of variation of global structural resistance, encompassing the influence of both material ($V_{R,m}$) and geometrical uncertainties ($V_{R,g}$); δ_R is the bias factor that adjusts for deviations in the estimation of $R_{NLNA}(f_m; a_n)$, considering both material ($\delta_{R,m}$) and geometrical ($\delta_{R,g}$) aspects. The following expressions can be employed to estimate, respectively, V_R and δ_R:

$$\delta_R = \delta_{R,m} \cdot \delta_{R,\mathrm{g}} \tag{3}$$

$$V_R = \sqrt{V_{R,\mathrm{m}}^2 + V_{R.g}^2} \tag{4}$$

Specific details can be acknowledged in [3].

Finally, β_T represents the target reliability index and α_R is the sensitivity factor of the first-order reliability method, assumed to be equal to 0.8 under the assumption of a dominant variable (*fib* Model Code for Concrete Structures 2010 [22]).

The primary objective of this study is to investigate the feasibility of establishing a simplified methodology for assessing $V_{R,m}$. To accomplish this, eight structural members, previously tested by other researchers and characterised by various failure mechanisms (including both nearly brittle and ductile), were selected from [27]-[29]. Subsequently, eight NLN models were created, calibrated and validated using the Cervenka Consulting ATENA 2D software. For each structural element, a probabilistic assessment of the structural response was conducted, employing a suitable probabilistic model for random material properties variables according to JCSS PMC 2001 [30]. Furthermore, the sensitivity of the probabilistic analysis to assumptions regarding concrete properties was considered, distinguishing between two values of the coefficient of variation (V_c) for the related cylinder compressive strength (i.e., 0.15 and 0.25).

The initial findings from the probabilistic analyses provide insights into discussing the variability of $V_{R,m}$ based on relevant structural response parameters, such as the maximum strain experienced within the reinforcement governing the failure mechanism.

2 Methods

In this investigation, an experimental dataset comprising eight structural members, originally designed and tested by Leonhardt & Walther (1966) [27], Foster & Gilbert (1998) [28] and Filho (1995) [29], is utilized. Table 1 offers an overview of the selected members, which includes five deep beams from Leonhardt & Walther (1966) [27], two from Foster & Gilbert (1998) [28] and one wall with an opening from Filho (1995) [29]. All specimens underwent construction in laboratory using a simply supported static setup.

Author	Structural member	R_{exp} [kN]	$f_{c,exp}$ [MPa]	Diameter [mm]	$f_{y,exp}$ [MPa]	$f_{u,exp}$ [MPa]
[27]	WT2	1085	28.7	8	419.9	536.6
	WT3	884	27.5			
	WT4	1670	26.7			
	WT6	990	28.3			
	WT7	1151	28.3			
[28]	B2.0A-4	1800	86.0	20	440	550
[29]	MB1ae	407	37.0	16	600	666
	MB1ee	413	42.0	10	530	670

Table 1: Key characteristics of the benchmark structural elements.

Table 1 details the findings of each experimental test, including the maximum load (R_{exp}), the compressive strength of the concrete cylinders ($f_{c,exp}$) and properties of the main reinforcement, such as bar diameter, yielding strength ($f_{y,exp}$) and ultimate strength ($f_{u,exp}$). For more detailed information on aspects like reinforcement layout, dimensions and structural member geometry, references [27]-[29] can be consulted. Figure 1 depicts the main features of the considered RC members. In this investigation, the terminology “governing reinforcement” refers to the tension bars predominantly implicated in the failure mechanism, reaching maximum strain at the point of failure.

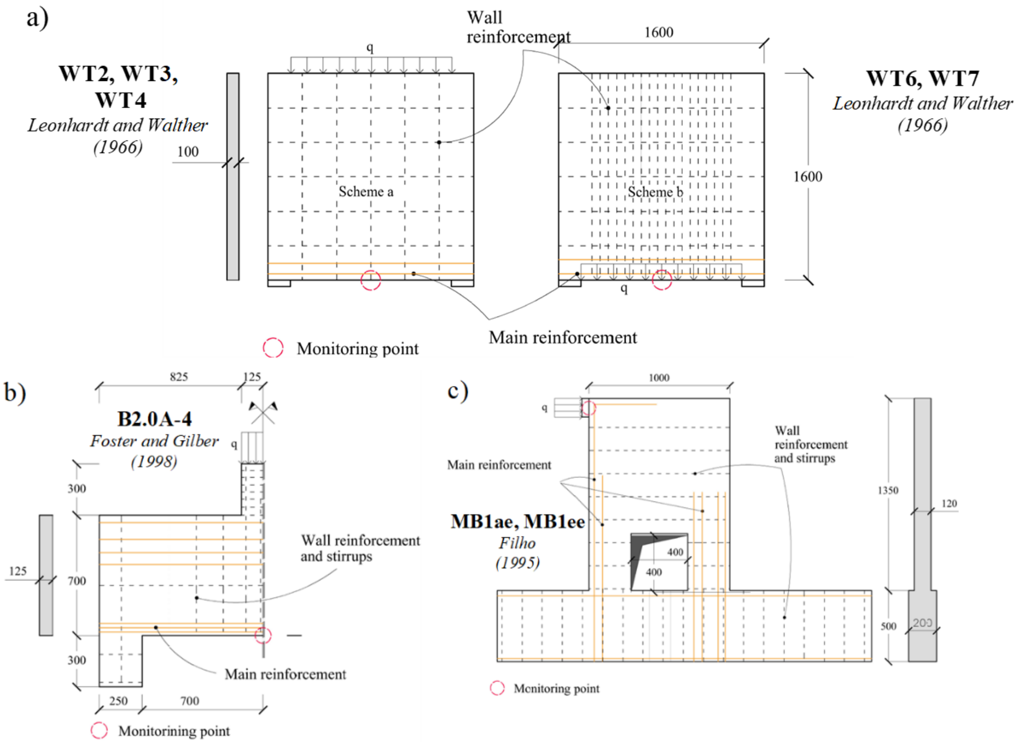


Figure 1: Depiction of the chosen structural elements and identification of the dominant reinforcement. Dimensions in centimeters.

In the following, the fundamental details regarding the establishment of the NLN models, their validation against experimental data and subsequent assumptions for probabilistic modelling of aleatory uncertainties are reported.

As first, the modelling assumptions [31] used to develop the NLN models for the previously described eight structural members are described. The ATENA 2D software [32] platform was employed for modelling. The main body of the RC members was represented using quadrilateral plane stress finite elements known as CCQ10SBeta [32], featuring quadratic displacement interpolation functions (ATENA 2D [32]). The size of the finite element mesh was determined through a calibration process for each structural member, ranging between 5 and 10 cm. The system of nonlinear equations was solved using the standard Newton-Raphson iterative procedure (*fib* Bulletin 45 [33]), with a maximum iteration number set at 200. Convergence criteria were based on forces and energy, with tolerances set to 1% and 0.01%, respectively. Regarding the constitutive models employed, the nonlinear behaviour of concrete in compression was represented using the SBeta material model [32] (ATENA 2D), which accounts for compression softening with a progressive reduction of compressive strength. The smeared crack modelling approach was adopted using the rotated crack model (ATENA 2D [32]). Concrete properties were set in accordance with experimental data from [27]-[29] (Table 1). If data were unavailable from the original scientific research, missing parameters were adopted in compliance with EN1992-1-1 [24]. Concerning the reinforcement, a bilinear with hardening constitutive law was employed to replicate reinforcement behaviour in both compression and tension. Key properties were defined in agreement with experimental results from [27]-[29] (Table 1). The Young's modulus of steel reinforcements was assumed to be 200.000 MPa and the associated ultimate strain ε_u was set to 9% [3]. The reinforcement yielding strain ε_y was derived in agreement with prior data and assumptions. The reinforcement was modelled using both smeared (wall and shear reinforcement) and discrete approaches (main reinforcement) (ATENA 2D [32]). Numerical simulations followed the experimental loading process, initially applying the dead load and subsequently the incremental load until failure. The recorded displacements in the numerical simulation align with those illustrated in Figure 1. Additionally, the maximum strain reached within the main reinforcement at failure (last load step with numerical convergence), denoted as ε_s^*, has been recorded for each NLN simulation.

The comparison between the experimental ultimate load R_{test} and the results of NLNA conducted using the experimental values (considered as mean values) of material properties $R_{sim,m}$ is depicted in Figure 2. As per [31], the ratio $\vartheta=R_{test}/R_{sim}$ represents the modelling uncertainty associated with the specific set of modelling assumptions.

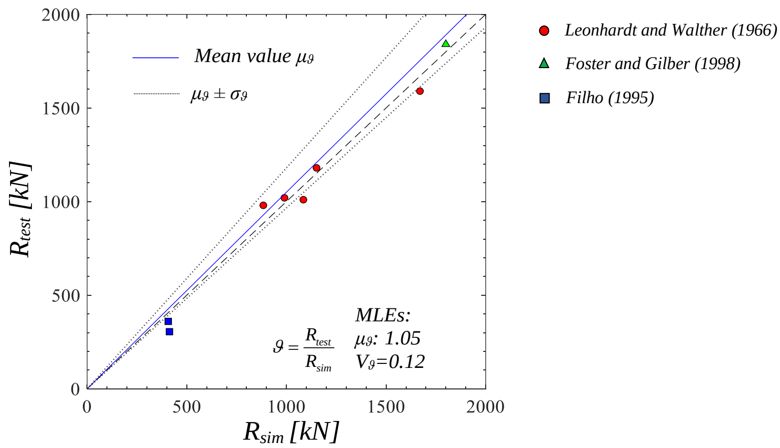


Figure 2: Comparison between tests and numerical results for the selected modelling assumptions.

The statistical characterization of the variable ϑ has been finalised through the maximum likelihood method, assuming a lognormal distribution with mean value μ_ϑ and coefficient of variation V_ϑ [17],[31]. Based on all the obtained results, μ_ϑ and V_ϑ were determined: 1.05 (indicating safely biased models) and 0.12, respectively. Consistent with prior investigations ([17]-[18]), it can be concluded that the modelling assumptions employed in this study are suitable for undertaking a probabilistic examination of structural response.

As second, the basic assumptions guiding probabilistic modelling to run probabilistic analysis of structural response, conducted through the Latin Hypercube Sampling method (LHS), in accordance with JCSS PMC 2001 [30] and *fib* Model Code for Concrete Structures 2010 [22] are presented. The primary aim of the study is to investigate the influence of aleatory uncertainties associated with material properties, specifically, focusing on characterizing the coefficient of variation $V_{R,m}$. To achieve this, the random variables outlined in Table 2 are adopted. While all geometrical parameters are assumed to be deterministic, properties that depend on the random variables in Table 2 are determined following EN1992-1-1:2014 [24] (e.g., Young's modulus and tensile strength of concrete) for each sample. The probabilistic model adheres to the specifications of JCSS PMC 2001 [30] and incorporates linear correlation among the relevant random variables. It is worth noting that the mean values of the various properties are aligned with experimental data [31]. Furthermore, two different values of the coefficient of variation V_c for concrete cylinder compressive strength (i.e., 0.15 and 0.25) have been considered to address the variability in material quality across different castings with respect to both new and existing RC systems [34].

Property	Distr. type	Mean value	Coefficient of variation [-]	Linear correlation coefficient*
f_c [MPa]	Lognormal	$f_{c,exp}$	0.15 – 0.25	-
f_y [MPa]	Lognormal	$f_{y,exp}$	0.05	f_u (0.85), ε_u(-0.50)
f_u [MPa]	Lognormal	$f_{u,exp}$	0.05	f_y (0.85), ε_u(-0.55)
E_s [MPa]	Lognormal	210000	0.03	-
ε_u [-]	Lognormal	0.09	0.09	f_y (-0.50), f_u (-0.55)
* (-) linear correlation coefficients related to diverse random variables.				

Table 2: Main random variables adopted to represent the variability of material properties (aleatory uncertainties) [30].

Building upon previous studies [4], it is reasonable to assume that 30 samples generated using the LHS method are sufficient, provided that the global coefficient of variation for the variables involved and the global response remains below 0.3, with an allowable error margin of within 5%.

3 Results

The following section delves into the primary findings of the probabilistic investigation.

The evaluation by probabilistic analysis has led to the characterization of structural response under various combinations of material properties [4],[35].

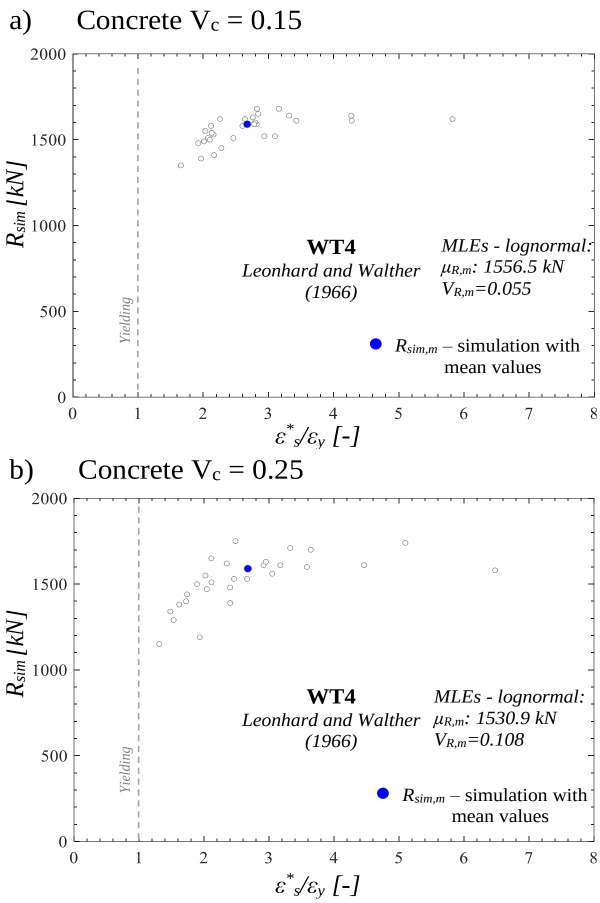
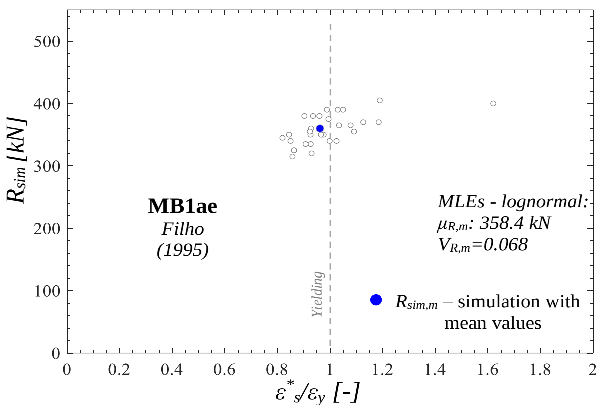


Figure 3: Outcomes in terms of R_{sim} for the deep beam WT4 from Leonhardt & Walter (1966) [27], considering different values for the coefficient of variation V_c.

The random variable representing structural resistance R, characterized by maximum likelihood estimators of a lognormal distribution [3], determines the mean value $\mu_{R,m}$ and coefficient of variation $V_{R,m}$ for each examined structural member. Typically, the failure mechanism corresponding to the simulation with mean values of material properties (i.e., experimental ones), denoted as $R_{sim,m}$, is the most probable when considering multiple failure modes. For instance, Figure 3(a)-(b) and Figure 4(a)-(b) illustrate the results of the WT4 [27] and MB1ae [29] structural members, respectively, with assumptions for V_c set to 0.15 and 0.25. Similar observations hold for other structural members, where the results from probabilistic simulations influence the value of the coefficient of variation $V_{R,m}$. This data can be integrated into safety evaluations using an appropriate format based on the global resistance method [22].

In the next, the insights gained from the probabilistic analysis of the RC members, with particular reference to the coefficient of variation $V_{R,m}$, are presented as a function of relevant structural response parameter. This parameter is expressed by the ratio $\varepsilon^*_s/\varepsilon_y$. Tracking the ratio $\varepsilon^*_s/\varepsilon_y$ during the NLNA conducted with mean values of material properties and nominal for geometrical ones offers a direct approach to characterize the structural response, providing information about the nature of the failure mechanism, whether ductile or brittle. Consequently, this parameter is suitable for establishing a general rule to determine $V_{R,m}$ in practical applications.

a) Concrete V_c = 0.15



b) Concrete V_c = 0.25

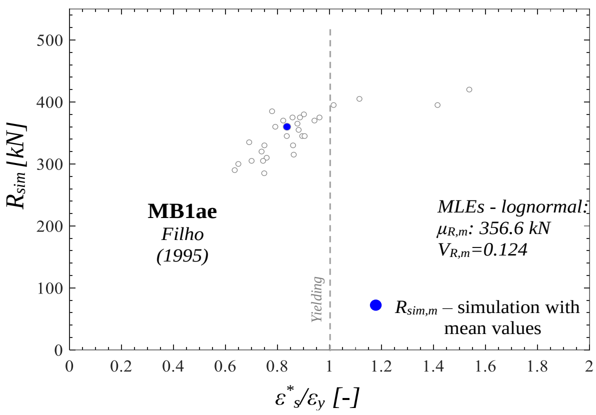


Figure 4: Outcomes in terms of R_{sim} for the wall MB1ae from Filho (1995) [29], considering different values for the coefficient of variation V_c.

Figures 5(a)-(b) and Figures 6(a)-(b) present the results regarding $V_{R,m}$ and the ratio between the mean value $\mu_{R,m}$ of structural resistances, obtained from the probabilistic analysis, and the structural resistance $R_{sim,m}$, achieved through a simulation using the mean values of material properties, related, respectively, to the two values for V_c (i.e., 0.15 and 0.25). The ratio $\varepsilon^*_s/\varepsilon_y$ pertains to results associated with the simulation using mean material (i.e., experimental) properties values and nominal for geometrical ones. Figures 5(a) and Figure 6(a) clearly illustrate the relationship between $V_{R,m}$ and the ratio $\varepsilon^*_s/\varepsilon_y$, and thus, the failure mechanism.

For $\varepsilon^*_s/\varepsilon_y < 1$, $V_{R,m}$ exhibits significant variability and, for very low values, approaches the coefficient of variation assumed for concrete compressive strength according to Table 2, indicating a brittle failure mechanism dominated by concrete failure. As the ratio $\varepsilon^*_s/\varepsilon_y$ increases, $V_{R,m}$ progressively decreases, approaching the coefficient of variation of the main reinforcement yielding strength, specifically, for values of $\varepsilon^*_s/\varepsilon_y$ higher than 5 for V_c=0.15 and much higher than 10 for V_c=0.25. This indicates a failure mechanism increasingly governed by the yielding of the main reinforcement. As the value of V_c increases, indicating lower concrete quality, higher ductility is required to minimize the value of $V_{R,m}$ concerning the global response.

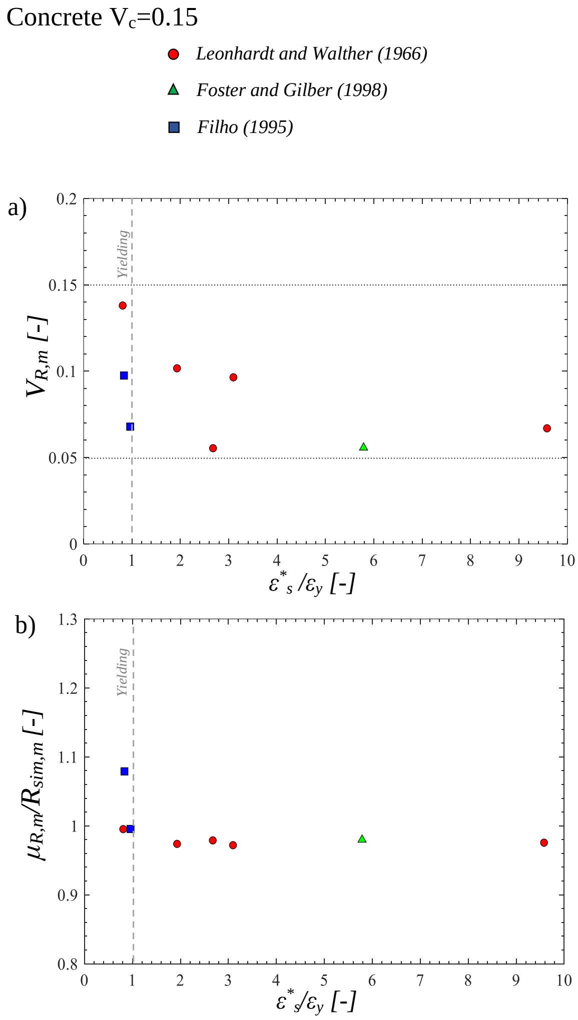


Figure 5: Results in terms of $V_{R,m}$ (a) and ratio $\mu_{R,m}/R_{sim,m}$ (b) dependent on the ratio $\varepsilon^*_s/\varepsilon_y$ – V_c=0.15.

Figure 5(b) and Figure 6(b) demonstrate the relationship between the ratio $\mu_{R,m}/R_{sim,m}$ and the ratio $\varepsilon^*_s/\varepsilon_y$. It is evident, consistent with the findings of [3], that this ratio can be reliably assumed to be equal to 1 for practical applications. For low values of $\varepsilon^*_s/\varepsilon_y$, the ratio $\mu_{R,m}/R_{sim,m}$, on average, exceeds one and approaches one for high values of $\varepsilon^*_s/\varepsilon_y$. This suggests that the numerical simulation conducted with mean values of material properties $R_{sim,m}$ provides a conservative estimate of $\mu_{R,m}$. These preliminary findings imply that, once $V_{R,m}$ is determined in accordance with the global resistance method (GRM), it is feasible to ascertain the partial safety factor γ_R and the design value of structural resistance according to Eq. (1) and Eq. (2).

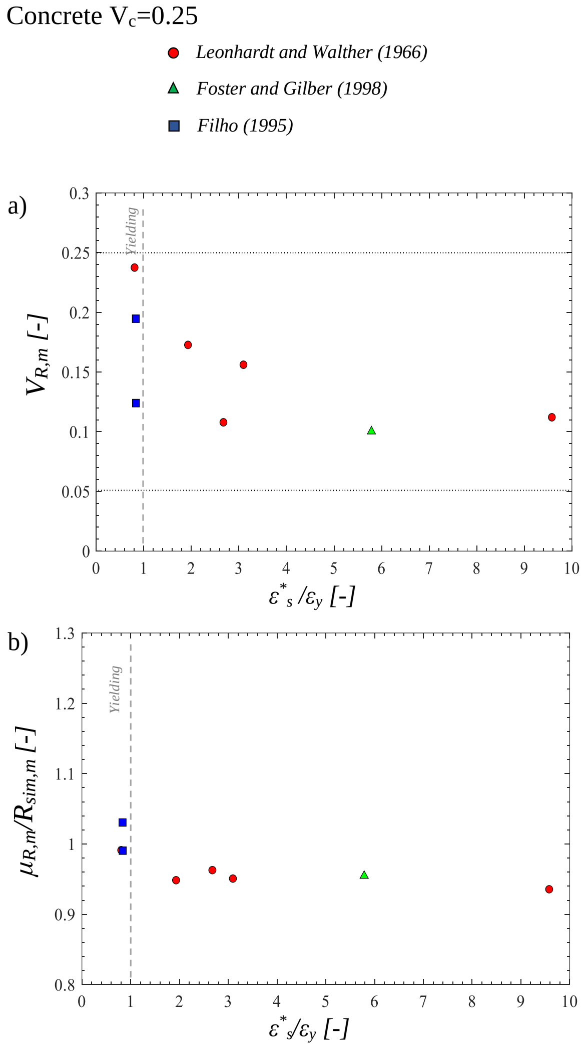


Figure 6: Results in terms of $V_{R,m}$ (a) and ratio $\mu_{R,m}/R_{sim,m}$ (b) dependent on the ratio $\varepsilon^*_s/\varepsilon_y$ – V_c=0.25.

4 Conclusions and Contributions

This research delves into the probabilistic characterization of structural response in RC members, with the aim of proposing improvements in safety formats for nonlinear analysis of RC structures. Specifically, the variability of the coefficient of variation $V_{R,m}$ of global structural resistance has been explored, considering the influence of randomness in material properties, including two values for concrete cylinder compressive strength coefficient of variation (i.e., 0.15 and 0.25). This variability has been correlated with the maximum strain observed in the main reinforcement within a numerical simulation conducted with mean values of material properties and nominal for geometrical ones. This initial correlation provides practical utility by enabling a direct estimation of $V_{R,m}$, thus reducing the required NLNAs needed to characterize the design value of structural resistance (R_d) in accordance with desired reliability levels.

To further strengthen the robustness and applicability of this approach, additional investigations are warranted. The future studies should validate and generalize the method across different probabilistic assumptions related to random material properties variability. Moreover, it is essential to extend the analysis to include various structural members exhibiting diverse failure mechanisms. This broader scope will contribute to a more comprehensive understanding of the proposed methodology and its potential application in different structural configurations.

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