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Modal Analyses by Eigenvector and Ritz Vector Methods

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Abstract

This paper delves into the practical implications of two modal dynamic analysis methods: eigenvector modal analysis and Ritz vector modal analysis. Eigenvector modal analysis, a staple in engineering practice, provides crucial insights into the dynamic behaviours of structures during free vibrations and forms the basis for modal superposition dynamic response analysis. Ritz vector modal analysis, primarily used for modal superposition dynamic response analysis, is examined for its unique characteristics and advantages. We emphasize the practical relevance of mode shapes and participation factors and demonstrate the load dependent nature of participation factors. The intricacies of Ritz analysis are explored, particularly highlighting its load dependent nature and the non-orthogonality of mode shapes for the applied loads. The essential steps and formulas for conducting Ritz analysis are presented, and practical recommendations for its uses in modal superposition dynamic response analysis are offered. We conclude by assessing the efficacy of Ritz vector modal analysis in modal superposition dynamic response analysis, underscoring its efficiency and potential benefits for engineering applications.

Keywords: modal analysis, participation factor, dynamic response, eigenvector, Ritz vector, modal superposition

1 Introduction

In structural engineering practice, modal analysis is widely used to investigate dynamic characteristics of structures, such as natural frequencies and mode shapes.

Modal analysis is also needed in dynamic response studies where the modal superposition dynamic response (MSDR) method is used.

Eigenvector modal analysis is load independent and focuses on studying the dynamic properties of structures under free vibrations. On the other hand, the MSDR analysis, including response spectrum analysis, harmonic analysis, footfall analysis, etc., studies dynamic responses of structures subjected to specific loads (represented by a load vector) [1]. The inefficiency of the MSDR analysis can arise if it uses the eigenvector modal analysis results, which produce many modes that are orthogonal to the load vector. These orthogonal modes do not contribute to the dynamic response and lead to redundant calculations. Ritz vector modal analysis [2], referred to as "Ritz analysis" in this paper, can be used to address this inefficiency. Like eigenvector modal analysis, Ritz analysis also gives natural frequencies and mode shapes, but its mode shapes are not orthogonal to the given load vector. Consequently, redundant calculations can be prevented by utilising Ritz analysis results in the MSDR analysis, thereby enhancing the efficiency and effectiveness of the analysis.

This paper aims to explore the two modal analysis methods, examine their differences, and present suitable applications for each of them. Additionally, an explanation of the participation factors, reflecting the relative contribution of each mode to the total dynamic response under the action of the loads considered, is given – a topic that engineers sometimes misunderstand.

2 Eigenvector modal analysis

Eigenvector modal analysis studies the dynamic properties of structures under free vibration. Its results are used to (i) understand the structure's dynamic properties and (ii) provide input for dynamic response analysis using the MSDR method.

Equation (1) is the governing dynamic equation [1], with the left hand side terms representing the inertia force, damping force, and restoration force from element deformations, respectively. The right hand side contains the applied load, expressed by the product of load vector and time variation function. This is not a general form of dynamic load, but as we will see later, this type of dynamic load is necessary to calculate modal participation factors, as well as the dynamic responses, using the MSDR method.

$$[M]\{\ddot{u}\} + [C]\{\dot{u}\} + [K]\{u\} = \{P\}f(t) \tag{1}$$

In eigenvector modal analysis, damping tends to be ignored because its effects on frequencies and mode shapes are very small, which is the case with almost all engineering problems. Also, as mentioned above, eigenvector modal analysis studies free vibration with no loads present. Considering these two factors, the governing equation for the eigenvector modal analysis is given by [2].

$$[K][\Phi] = [M][\Phi][\Omega] \tag{2}$$

This is an eigenvalue problem; the unknowns are the eigenvalue matrix $[\Omega]$ and the eigenvector matrix $[\Phi]$. Many solution methods are available, such as the Jacobian method for solving all eigenvalues & eigenvectors and the subspace iteration method for solving the first few eigenvalues & eigenvectors. The solution to this equation is not discussed here but can be found in [1] or any other dynamic analysis books. Here, we assume the eigenvalue and eigenvector matrices, $[\Omega]$ and $[\Phi]$ have been found and are expressed as

$$[\Omega] = \begin{bmatrix} \omega_1^2 & 0 & 0 & 0 \\ 0 & \omega_2^2 & 0 & 0 \\ 0 & 0 & \dots & 0 \\ 0 & 0 & 0 & \omega_m^2 \end{bmatrix} \tag{3}$$

$$[\Phi] = [\{\varphi_1\} \quad \{\varphi_2\} \quad \dots \quad \{\varphi_m\}] = \begin{bmatrix} \varphi_{11} & \varphi_{12} & \cdots & \varphi_{1m} \\ \varphi_{21} & \varphi_{22} & \cdots & \varphi_{2m} \\ \cdots & \cdots & \cdots & \cdots \\ \varphi_{m1} & \varphi_{m2} & \cdots & \varphi_{mm} \end{bmatrix} \tag{4}$$

where:

ω_i – circular frequency of mode *i* $\quad$ *i = 1, 2, 3…m*
$\{\varphi_i\}$ – eigenvector of mode *i* $\quad$ *i = 1, 2, 3…m*

2.1 Characteristics of eigenvectors

(i) Eigenvectors are orthogonal to each other with respect to the mass matrix, as shown in equation (5), in which the transpose of one eigenvector multiplied by the mass matrix and then by the same eigenvector is equal to the modal mass for the same mode. The transpose of one eigenvector multiplied by the mass matrix and then by a different eigenvector equals zero.

$$[\Phi]^T[M][\Phi] = \begin{bmatrix} m_1 & 0 & 0 & 0 \\ 0 & m_2 & 0 & 0 \\ 0 & 0 & \dots & 0 \\ 0 & 0 & 0 & m_m \end{bmatrix} \tag{5}$$

(ii) Eigenvectors are orthogonal to each other with respect to the stiffness matrix, as shown in equation (6), where k_1, k_2 , etc. represent modal stiffness.

$$[\Phi]^T[K][\Phi] = \begin{bmatrix} k_1 & 0 & 0 & 0 \\ 0 & k_2 & 0 & 0 \\ 0 & 0 & \dots & 0 \\ 0 & 0 & 0 & k_m \end{bmatrix} \tag{6}$$

(iii) Eigenvectors are orthogonal to each other with respect to the damping matrix. This orthogonality is conditional, and the use of classical damping is required. Classical damping, $[C]$, is defined as a linear combination of the stiffness and mass matrices, as shown in equation (7):

$$[C] = a[M] + b[K] \tag{7}$$

where:

a – stiffness proportional damping coefficient
b – mass proportional damping coefficient

In this case, the eigenvectors will have a generalised orthogonality with respect to $[C]$. The use of classical damping is required for decoupling the multiple degrees of freedom (DOF) system into the required number of single DOF systems, which provides the basis for the MSDR analysis. In the following sections, classical damping is assumed throughout.

2.2 Dynamic equations in modal coordinates

Assuming eigenvectors are known, they can be taken as a new coordinate system, i.e. the modal coordinate system in eigenvector space, and the dynamic equation (1) in the global coordinate system can be transformed to this modal coordinate system. Assuming $\{q\}$, $\{\dot{q}\}$, and $\{\ddot{q}\}$ are the displacement, velocity and acceleration vectors in the modal coordinate system, respectively. The original nodal displacement, velocity, and acceleration vectors can be expressed in this modal coordinate system as:

$$\begin{aligned} \{u\} &= [\Phi]\{q\} \\ \{\dot{u}\} &= [\Phi]\{\dot{q}\} \\ \{\ddot{u}\} &= [\Phi]\{\ddot{q}\} \end{aligned} \tag{8}$$

The substitution of equation (8) into equation (1) gives:

$$[M][\Phi]\{\ddot{q}\} + [C][\Phi]\{\dot{q}\} + [K][\Phi]\{q\} = \{P\}f(t) \tag{9}$$

Multiplying equation (9) by the transpose of the eigenvector matrix from the left gives:

$$[\Phi]^T[M][\Phi]\{\ddot{q}\} + [\Phi]^T[C][\Phi]\{\dot{q}\} + [\Phi]^T[K][\Phi]\{q\} = [\Phi]^T\{P\}f(t) \tag{10}$$

Using the orthogonal characteristics of eigenvectors with respect to mass, damping, and stiffness matrices, equation (10) can be reduced into m uncoupled single DOF equations:

$$\{\varphi\}_i^T[M]\{\varphi\}_i\{\ddot{q}\} + \{\varphi\}_i^T[C]\{\varphi\}_i\{\dot{q}\} + \{\varphi\}_i^T[K]\{\varphi\}_i\{q\} = \{\varphi\}_i^T\{P\}f(t) \quad (i = 1, 2\ldots\text{m}) \tag{11}$$

Let:

$$m_i = \{\varphi\}_i^T[M]\{\varphi\}_i \quad c_i = \{\varphi\}_i^T[C]\{\varphi\}_i \quad k_i = \{\varphi\}_i^T[K]\{\varphi\}_i \tag{12}$$

Where:

m_i, c_i and k_i are modal mass, modal damping, and modal stiffness, respectively for mode i.

Then m uncoupled single DOF equations can be simplified as:

$$m_i\ddot{q}_i + c_i\dot{q}_i + k_iq_i = \{\varphi\}_i^T\{P\}f(t) \qquad (i = 1, 2, \ldots\text{m}) \tag{13}$$

Which can be further simplified to:

$$\ddot{q}_i + 2\xi_i\omega_i\dot{q}_i + \omega_i^2 q_i = \Gamma_i f(t) \qquad (i = 1, 2, \dots \text{m}) \tag{14}$$

Where:

$2\xi_i\omega_i = \frac{c_i}{m_i}$ $\qquad$ $\omega_i^2 = \frac{k_i}{m_i}$

$\Gamma_i = \frac{\{\varphi\}_i^T\{P\}}{m_i}$ modal participation factor (discussed in the next section)

ξ_i critical damping ratio for mode *i*

ω_i circular frequency for mode *i*

2.2.1 Summary of main features of the dynamic equation in the modal coordinate system

- The multiple DOF dynamic equations are simplified to m uncoupled single DOF equations.
- The m uncoupled single DOF equations are in the same form as that shown in equation (14).
- Only one equation needs to be solved numerically or analytically to obtain the general solution.
- The solutions for other equations/modes can be easily obtained by factorising the general solution by the relevant modal participation factor Γ.

2.2.2 Participation factors

- Participation factors for imposed load:

  The participation factor Γ of mode *i* is equal to the product of the transpose of the mode shape vector and the load vector, divided by the modal mass, m_i.

$$\Gamma_i = \frac{\{\varphi\}_i^T\{P\}}{m_i} \qquad i = 1, 2 \dots \text{m} \tag{15}$$

- Participation factors for ground acceleration load:

  The equivalent imposed load due to ground acceleration is [1]:

$$\{P\}f(t) = -[M]\{r\}\alpha(t) \tag{16}$$

  Where:

  $\{r\}$ – displacement transformation vector [3] that gives the active degree of freedom due to ground acceleration.

  $\alpha(t)$ – ground acceleration.

  According to equation (16), the participation factor for mode *i* is:

$$\Gamma_i = \frac{\{\varphi\}_i^T\{P\}}{m_i} = \frac{\{\varphi\}_i^T[M]\{r\}}{m_i} \qquad i = 1,2, \dots \text{m} \tag{17}$$

- Characteristics of participation factors

a. Participation factors reflect the relative contribution of each mode to the total dynamic response under the action of the loads considered.
b. Participation factors are load dependent; the associated load vector should always be remembered when discussing and using the participation factors.
c. Participation factors can be defined only when the dynamic loads can be decoupled into the product of a load vector $\{P\}$ and a time variation function f (t). A load vector represents the spatial distribution of the loads, and the time function gives the variation of the loads with time.
d. The more the mode shape vector $\{\varphi\}$ is parallel to the load vector $\{P\}$, the larger the participation factor for that mode, and vice versa.
e. If a mode shape is orthogonal to the load vector $\{P\}$, the participation factor for that mode is zero, i.e. this mode will make no contribution to the total dynamic response from that load.
f. Participation factors given by most dynamic analysis software are for ground acceleration loads, so they can be used only for ground acceleration dynamic response analysis.
g. For a general imposed load case, participation factors, Γ_i , nodal displacement, $\delta_{i.j}$, nodal mass, M_j, and applied nodal loads p_j should satisfy the following equation:

$$M_j \sum_{i=1}^{m} \delta_{i,j} \Gamma_i = p_j \qquad (j = 1, 2, 3 \ldots \text{n – node number})$$

Where: $\delta_{i,j}$ is node j displacement in mode i

h. For a ground acceleration load case, participation factors and the modal displacements should satisfy this equation.

$$\sum_{i=1}^{m} \delta_{i,j} \Gamma_i = 1.0 \qquad (j = 1, 2, 3 \ldots \text{n – node number})$$

3 Modal Superposition Dynamic Response Analysis

Before delving into the Ritz analysis, the analysis procedures and characteristics of MSDR analysis are summarised below, which helps us understand the benefit of using Ritz analysis in MSDR analysis.

3.1 Procedures of MSDR analysis

- Carry out a modal dynamic analysis to obtain the frequencies, mode shapes and participation factors of the model.
- Transform the governing dynamic equations into the modal coordinate system and decouple the multiple DOF equation into m uncoupled single DOF equations.
- Solve one of the m single DOF equations to get the dynamic response of that mode.

- Obtain the response for all other modes by appropriately factorising the responses obtained above by the relevant participation factors and frequency related factors.
- The total dynamic response is the sum of the responses from all the modes. This sum depends on the type of analysis being carried out, such as the arithmetic sum, in the case of time history analysis, or SRSS (Square Root of the Sum of the Squares) and CQC (Complete Quadratic Combination) [1], in the case of steady state dynamic response analysis, such as harmonic and response spectrum seismic analyses.

3.2 MSDR analysis results

- As the total dynamic response is the sum of the responses from all modes considered, the more modes are considered, the better the results. However, as higher mode contributions become progressively smaller, they are often ignored in practice.
- Each mode's contribution to the total response is related to its participation factor; the larger the participation factor, the bigger the contribution.
- Each mode's dynamic response depends on its modal frequency and the load's dominant frequency. If modal and load frequencies are close, the response will be large.
- The dynamic response of each mode also depends on its modal damping; obviously, the higher the modal damping, the lower the contribution this mode makes.
- If a mode shape is orthogonal to the load vector, i.e. the participation factor $\Gamma = 0$, this mode will make no contribution to the total dynamic response
- If many modes are orthogonal or closed to orthogonal to the load vector, the MSDR analysis using eigenvector modal analysis results will be inefficient, as many redundant calculations are involved.
- To have reliable results from the MSDR analysis, the total participation masses should be higher than a certain level, e.g. ≥ 90% required by most design codes. To achieve the required participation mass for a large model, the total number of modes required could be very large, even exceeding computer capacity and making the analysis impractical.

4 Ritz Analysis

Ritz analysis gives the same type of results as the eigenvector modal analysis in terms of modal frequencies and mode shapes. The main difference between them is that the Ritz analysis considers the applied loads, while the eigenvector modal analysis does not. Ritz analysis works in the Ritz vector space, generated from the load vector in such a way as to guarantee that none of the modes are orthogonal to the load vector.

4.1 The Ritz analysis – general procedure

- To generate Ritz vectors that are not orthogonal to the load vector, a subspace called Ritz vector space is created with a dimension equal to the required number of modes.
- The stiffness matrix $[K]$ and mass matrix $[M]$ are transformed from the eigenvector space to the Ritz vector space, as explained in the next section. Since the dimension of the Ritz vector space is much smaller than the original eigenvector space, the total number of DOF of the system is significantly reduced.
- The reduced eigenvalue problem in the Ritz vector space is solved directly to obtain the eigenvalues (frequencies) and the eigenvectors (mode shapes) in the Ritz vector space.
- The mode shapes from the Ritz vector space are then transformed back to the eigenvector space to have the normal eigenvectors (mode shapes).
- The eigenvalues are the same in the Ritz vector space and the eigenvector space, so no transformation is necessary.
- After obtaining the frequencies and mode shape, the participation factors can be calculated using equations (15) or (17) given above.
- Once all the results are obtained, the analysis is completed.

4.2 Steps in the Ritz analysis

- Generate Ritz vectors $\{x_i\}$ from a given load vector $\{P\}$, $i = 1, 2, 3 \ldots$ R, where R is the number of the Ritz vectors (modes) required. R is governed by the required participation mass, and it is normally much smaller than the number of eigenvector modes

The first Ritz vector:

Calculate the displacement vector from the given load.

$$\{y_1\} = [K]^{-1}\{P\} \tag{18}$$

Normalise the displacement vector, which gives the first Ritz vector $\{x_1\}$.

$$\{x_1\} = \frac{\{y_1\}}{\sqrt{\{y_1\}^T[M]\{y_1\}}} \tag{19}$$

The second and the following Ritz vectors:

Take the product of the mass matrix and the previous Ritz vector as the fictitious load vector, then use it to calculate the displacement vector.

$$\{y_i^*\} = [K]^{-1}[M]\{x_{i-1}\} \tag{20}$$

Make the displacement vector orthogonal to other Ritz vectors.

$$\{y_i\} = \{y_i^*\} - \sum_{j=1}^{i-1}\{x_j\}^T[M]\,\{y_i^*\}\{x_j\} \tag{21}$$

Normalise the orthogonalised displacement vector, which gives the next Ritz vector [x_i]

$$\{x_i\} = \frac{\{y_i\}}{\sqrt{\{y_i\}^T[M]\{y_i\}}} \qquad i = 2, 3, \ldots R \qquad (22)$$

- Express the eigenvector space eigenvector matrix by the Ritz vector space eigenvector matrix

  The Ritz vector matrix is composed by the R Ritz vectors:

$$[X] = [\{x_1\}\,\{x_2\}\ldots\{x_R\}] \qquad (23)$$

  Let $[\Phi^*]$ represent the eigenvector matrix in the Ritz vector space, then the eigenvector matrix $[\Phi]$ in the eigenvector space can be expressed as:

$$[\Phi] = [X][\Phi^*] \qquad (24)$$

- Substitute equation (24) into the standard eigenvalue problem equation (2) gives:

$$[K][X][\Phi^*] = [M][X][\Phi^*][\Omega] \qquad (25)$$

  Pre-multiplying equation (25) by the transpose of the Ritz vector matrix $[X]^T$ gives:

$$[X]^T[K][X][\Phi^*] = [X]^T[M][X][\Phi^*][\Omega] \qquad (26)$$

  Let

$$[K^*] = [X]^T[K][X]$$
$$[M^*] = [X]^T[M][X]$$

  Then, the eigenvalue problem in the Ritz vector space is simplified as equation (27), and it is the same as equation (2):

$$[K^*][\Phi^*] = [M^*][\Phi^*][\Omega] \qquad (27)$$

- Solve equation (27) in the same way as equation (2); the eigenvectors and eigenvalues in Ritz vector space are obtained.

As the dimension R is very small, the Jacobean method is used to solve for all the eigenvectors $[\Phi^*]$ and eigenvalues $[\Omega]$. Eigenvalues $[\Omega]$ are the same in both the eigenvector space and the Ritz vector space, while the eigenvector matrix $[\Phi^*]$ needs to be transformed back to the eigenvector space from:

$$[\Phi] = [X][\Phi^*] \qquad (28)$$

So now, all eigenvectors and eigenvalues in the eigenvector space are known, and the problem is solved.

4.3 Summary of the Ritz analysis

- Ritz analysis is load dependent, so a load vector is needed. As mentioned above, the mode shapes from the Ritz analysis will not be orthogonal to the given load vector.
- Ritz analysis can achieve more participation mass with fewer modes than is required by the eigenvector modal analysis.
- Ritz analysis is a direct method with much reduced DOF, so the analysis is generally much faster than the eigenvector modal analysis and requires less memory space.
- Ritz analysis is a constrained and approximate method, as the mode shapes are restrained to be in the Ritz vector space after transformation.
- Although Ritz analysis is approximate, it is more useful in some practical cases than the 'precise' eigenvector modal analysis, especially for large DOF problems.
- Because its results are load dependent, results from the Ritz analysis can be used in the MSDR analysis only when the loads are the same as those used in the Ritz analysis.
- The frequencies and mode shapes from the Ritz analysis may be influenced by the total number of modes analysed. For example, the first 6 modes may be different from two separate Ritz analyses of the same model if they consider different total number of modes. Because of this, a slightly higher number of modes than required is recommended.
- Generally, natural frequencies from the Ritz analysis are equal to or higher than the natural frequencies from the eigenvector modal analysis; this is because the Ritz analysis represents a constrained solution.
- The lower frequency results are more accurate compared with those from the eigenvector modal analysis, although some higher frequency results may not be, this does not have a significant effect on the MSDR analysis that uses Ritz analysis results, as higher mode contributions to the dynamic responses are very small, even negligible.
- If the Ritz vector analysis results are used in the MSDR analysis, and the loads to be analysed are not the same as the load vector used in the Ritz analysis, the MSDR results are approximate and should be used with caution.

4.4 When to use Ritz analysis

- In seismic analysis, when the required percentage of participation masses cannot be achieved in eigenvector modal analysis using a reasonable number of modes.
- When the model size is very large, and the load directions are known, e.g., when analysing the dynamic response to wind load, seismic load, footfall load, etc.

- When localised responses are of interest, but the localised mode cannot be obtained from the eigenvector modal analysis. This requires the software to allow the user to define the load vector for the Ritz analysis.

4.5 When not to use Ritz analysis

- For a general study of dynamic properties of structures, the eigenvector modal analysis is more suitable because the Ritz analysis does not give all possible modes.
- For small and medium size models, when the required participation mass can be achieved using a reasonable number of modes in the eigenvector modal analysis.
- If the Ritz analysis can only take *x*, *y* & z direction ground motion as input load vectors (a common case in most software), and the dynamic loads are not in these three directions, e.g. the twisting load about vertical axis.

4.6 Number of modes needed in the Ritz analysis

- Considering a given direction in the Ritz analysis, e.g. vertical direction, the number of modes considered should cover all possible modes in that direction, e.g. the symmetric and un-symmetric ones.
- The target number of modes in the Ritz analysis should give very high participation masses, e.g. 98% or higher, as Ritz analysis can achieve high participation masses with a relatively small number of modes.
- If it is unclear how many modes are needed to achieve the required participation mass, a trial-and-error approach can be used, as the Ritz analysis is fast.

4.7 The accuracy of MSDR analysis using Ritz analysis results

- In the MSDR analysis, if dynamic loads (e.g. seismic loads) are exactly the same as the load vector used in the Ritz analysis, the MSDR analysis results would be almost 100% accurate.
- If dynamic loads are similar to the load vectors used in the Ritz analysis, the analysis results from MSDR analysis are approximate but still usable in engineering practice; for example, footfall analysis uses the Ritz approach with the vertical load vector, and x-direction wind load response uses the Ritz approach with the x-direction load vector.
- If dynamic loads are orthogonal (or close to orthogonal) to the load vectors used in the Ritz analysis, the MSDR analysis results could be totally wrong.

5 Conclusions

- Eigenvector modal analysis is the general method of studying the dynamic properties of a structure.
- Participation factors are load dependent.

- In most software, participation factors are given for ground motions; if this is the case, they should not be used for other load cases.
- If many modes from eigenvector analysis are orthogonal (or close to orthogonal) to the load vector, the MSDR analysis using eigenvector analysis results is inefficient, as it would involve redundant calculations.
- Ritz analysis gives modes that are not orthogonal to the load vector considered.
- Ritz analysis results lead to a more efficient MSDR analysis with no redundant calculations compared to the eigenvector modal analysis.
- Ritz analysis does not give all possible natural modes, so eigenvector modal analysis should be used in general studies of structural dynamic properties.

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