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A Numerical Model for Thermal Buckling Analysis of Functionally Graded Porous Thin-Walled Structures

S. Kvaternik Simonetti, D. Lanc, G. Turkalj and D. Banić

**Faculty of Engineering, University of Rijeka
Croatia**

A numerical model to predict the thermal buckling analysis of thin-walled porous functionally graded (FG) beams is presented in this work. A geometric nonlinear algorithm that uses a 1D numerical model with a spatial beam finite element is employed. The Green-Lagrange deformation tensor defines small deformations. The Euler-Bernoulli theory for bending and the Vlasov theory for torsion are used to create the finite element model. Nonlinear analysis uses the UL (updated Lagrangian) incremental formulation with the principle of virtual works. The cross-sectional displacement field accounts for warping torsion and large rotations. Material properties are assumed to vary continuously through the wall thickness based on power-law distribution. The proposed beam model analyses buckling in cases of uniform, linear, and nonlinear temperature distribution through the thickness of the cross-sectional walls. The analysis also considers the temperature dependence of the mechanical properties of the material. Numerical results investigate critical buckling temperatures and post-buckling responses for different thin-walled sections with various configurations. These configurations include boundary conditions, geometry, FG skin-core-skin ratios, and power-law index. The numerical algorithm's accuracy and reliability are compared with established software packages' 2D finite element models. The comparison shows excellent agreement with the results obtained with shell models.

Keywords: numerical analysis, buckling and post buckling, thermal environment, functionally graded beam, thin-walled cross-section, porous material, temperature distribution.

1 Introduction

Functionally graded (FG) materials with uniform porosity distribution are a new class of advanced composite materials. Most often, FG materials are composed of ceramic and metal, and their material properties change continuously across the thickness of the cross-section. Compared to traditional composites, FGMs have several advantages such as high durability, toughness, thermal and corrosion resistance.

The thermal buckling and vibration of FG thin-walled beams and structures have become interesting topic for an increasing number of researches due to the complex behaviour of these lightweight structures with improved thermomechanical properties, but only a few of them are cited here [1–5]. However, in the case of buckling of FG porous beam in thermal environment, the literature is quite scarce [6–11].

The paper discusses a beam model for the thermal buckling of thin-walled porous beams made of functionally graded materials (FGMs). The model is based on the Euler-Bernoulli-Navier bending theory and Vlasov torsion theory while assuming large displacement and small strains. The equilibrium equations of the finite elements are developed using an updated Lagrangian formulation, and the Newton-Raphson method is used as an incremental iterative solution scheme. The material properties are assumed to be graded across the wall thickness, and three different cases of temperature rise over wall thickness are considered, which are uniform, linear, and nonlinear. The numerical results are obtained for FGM beams with different boundary conditions, porosity coefficient, and temperature distributions to investigate the effects of the power-law index on the critical buckling temperature and post-buckling response.

The main objective of the paper is to present the developed beam model for the thermal buckling analysis of FG thin-walled beam structures and to discuss the influence of the porous volume fraction buckling behaviour. The analysis is based on the numerical model developed by the authors [12–14] and verified by benchmark shell examples.

2 Methods

Consider an imperfect FG beam with evenly disperse porosities. The material properties vary continuously through the wall thickness according to the power law distribution [7]:

$$P\,(n,T) = [P_o\,(T) - P_i\,(T)] \cdot V_c\,(n) + P_i\,(T) - 0{,}5\rho \cdot [P_o\,(T) + P_i\,(T)]. \tag{1}$$

The equation shown above defines the effective material property, which is represented by the variable P. This property can be Young's modulus E, shear modulus G, or coefficient of thermal expansion α and conductivity K. The subscripts i and o represent the inner and outer surface constituents respectively. Additionally, V_c is the volume fraction of the ceramic phase, and there can be multiple variations of material distributions within the wall thickness. The small imperfection of the material is presented by scalar coefficient ρ<<1. Poisson's ratios v is assumed constant. Imperfect porous FG material is shown in Fig. 1.

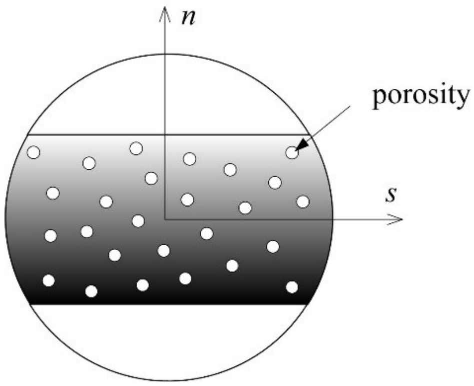

Figure 1: Imperfect porous FG material.

In order to predict the buckling behaviour of FG beams under thermal loads more accurately, material properties are considered temperature dependent. Nonlinear equation of material properties in function of temperature T(K) can be written as:

$$P = P_0 (P_{-1} T^{-1} + 1 + P_1 T + P_2 T^2 + P_3 T^3), \tag{2}$$

where temperature coefficients P_0, P_{-1}, P_1, P_2 and P_3 are unique for every material [15].

The relationship between stress and strain can be expressed using the generalized form of Hooke's law, which states that:

$$\sigma_z = E\,(n,T) \cdot [\varepsilon_z - \alpha\,(n,T) \cdot \Delta T], \\ \tau_{zs} = G\,(n,T) \cdot \gamma_{zs}. \tag{3}$$

The stress components are denoted by σ_z and τ_{zs}, while the strain components are represented by ε_z, and γ_{zs}. The flange normal and transverse directions are indicated by n and s, respectively, while z is parallel to the beam axis. Additionally, ΔT refers to a temperature change.

A uniform, linear, and nonlinear temperature distribution is applied across the wall thickness of the beam. In the case of absence of heat generation, steady-state one-dimensional heat conduction equation can be written as [16]:

$$\mathrm{d}(K(n,T)\mathrm{d}T/\mathrm{d}n)/\mathrm{d}n = 0, \tag{4}$$

and the temperature distribution across the wall thickness can be expressed as:

$$T(n) = T_i(z) + C(T) \cdot [T_o(z) - T_i(z)] \,/\, D(n,T), \tag{5}$$

where C and D can be found in Ref [2]. Assuming equal coefficients of thermal conductivity $K_i = K_o$, equation (5) can be used to derive an expression for linear temperature distribution. However, to achieve uniform temperature distribution, it is necessary to assume uniform temperature of the beam, that is $T_o = T_i$.

3 Results

Consider a thin-walled FG Channel-section beam with the length $l = 4$ m, Fig. 2. The beam is subjected to uniform temperature distribution. Two cases of boundary conditions are analysed, clamped-clamped and simply supported, and three types of material distribution, C1, C2 and C3. The components of the FG material are Al_2O_3 (E_c = 346 GPa, $\alpha_c = 6.86 \cdot 10^{-6}$ 1/°C) as ceramic, and SUS304 (E_m = 207 GPa, $\alpha_m = 1.53 \cdot 10^{-5}$ 1/°C) as metal.

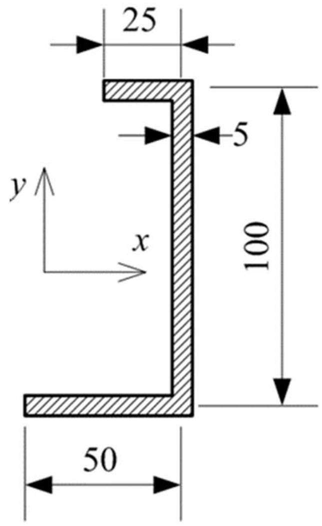

Figure 2: Channel-section beam.

FG material distribution C1 is composed of ceramic-rich bottom skin and FG skin at the top in the ratio 1:2. The volume fraction of the ceramic V_c can be given by:

$$V_c = 1, \quad t_0 \le n \le t_1,$$
$$V_c = [(n - t_3)/(t_1 - t_3)]^p, \quad t_1 \le n \le t_3. \tag{6}$$

Distribution C2 is made of ceramic core and FG skins in the ratio 1:1:1. The top skin varies from a ceramic-rich to a metal-rich surface, while the bottom skin varies from a ceramic-rich to metal-rich. V_c can be determined as follows:

$$V_c = [(n - t_0)/(t_1 - t_0)]^p, \quad t_0 \le n \le t_1,$$
$$V_c = 1, \quad t_1 \le n \le t_2,$$
$$V_c = [(n - t_3)/(t_2 - t_3)]^p, \quad t_2 \le n \le t_3. \tag{7}$$

Material distribution C3 is composed of FG core and ceramic-rich bottom skin and metal-rich top skin in the ratio 1:2:1. In this case, V_c can be expressed as:

$$V_c = 1, \quad t_0 \le n \le t_1,$$
$$V_c = [(n - t_1)/(t_2 - t_1)]^p, \quad t_1 \le n \le t_2,$$

$$V_c = 0, \quad t_2 \leq n \leq t_3. \tag{8}$$

The analysis of the thermal buckling of a beam was carried out for various values of the power-law index p. The results of the author's beam model were verified using a numerical model based on shell finite elements. The FG material was simulated by homogeneous layers.

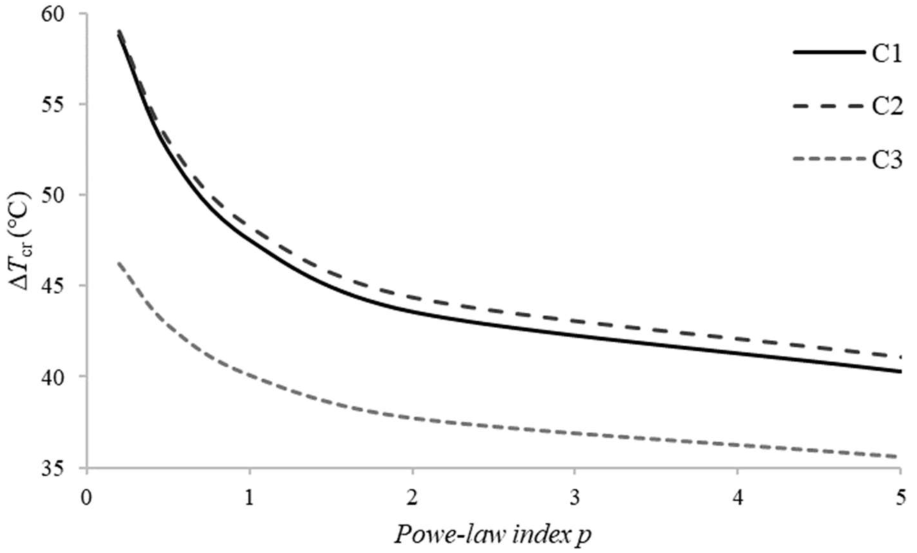

Figure 3: Critical buckling temperatures for different material distributions and ρ = 0.2.

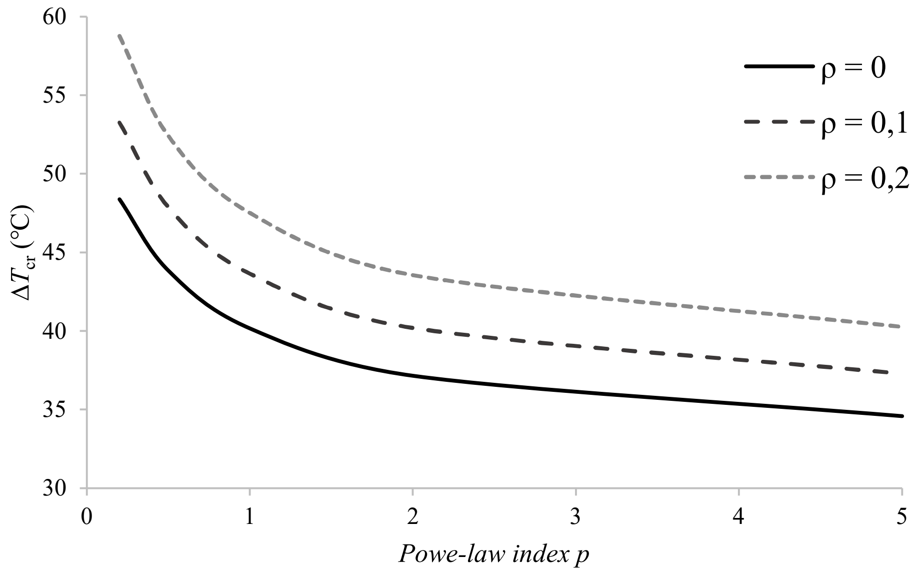

Figure 4: Critical buckling temperatures of C2 for different porosity coefficient ρ.

Fig. 3 illustrates a comparison of the critical buckling temperatures with different power-law values p and FG material distribution for porosity coefficient $\rho = 0.2$. The C3 distribution showed the lowest temperatures due to the higher proportion of metal in the cross-section. In Fig. 4, a comparison of buckling temperatures of the C2 for different porosity coefficients is shown. The results indicate that as the porosity coefficient increases, the critical buckling temperature also increases. This is because an imperfect beam is less thermally conductive than a perfect one.

A non-linear buckling analysis for porous materials was also performed and a comparison with temperature-dependent material properties was given. Below are the results for the two-sided clamped beam of C2 and C3 distributions for the exponent $p = 0.2$ and the porosity coefficient $\rho = 0.1$. It is also show comparison between temperature independent (TID) and temperature dependent (TD) material properties. The diagram shows a good agreement between the nonlinear response curves and the eigenvalues calculated using shell model. As expected, TD materials reach lower buckling temperature.

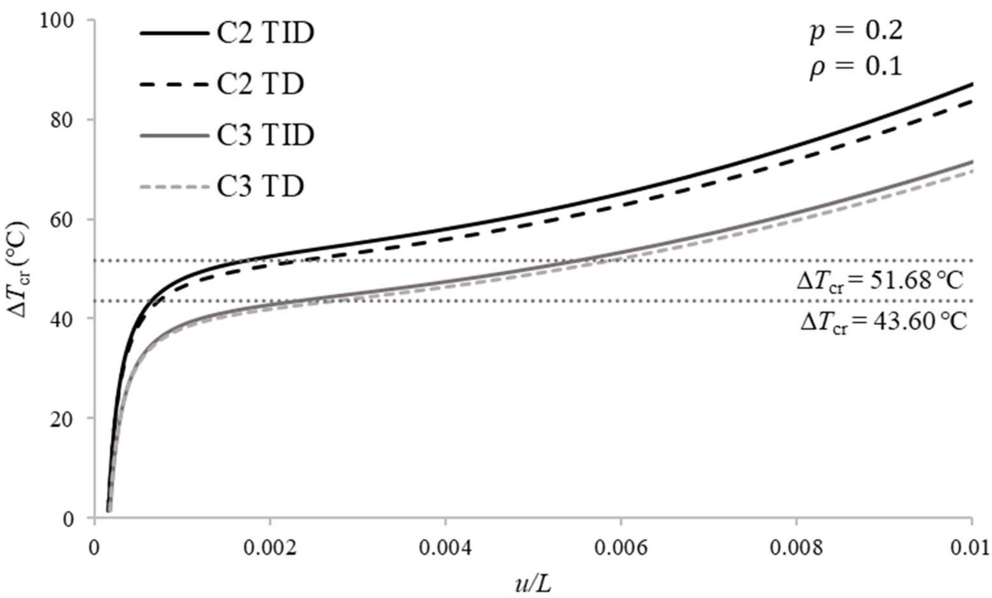

Figure 5: Critical buckling temperatures of C2 for different porosity coefficient ρ.

4 Conclusions and Contributions

A numerical model has been developed to analyse the thermal buckling of a thin-walled FG porous beam. The model uses finite element incremental equilibrium equations that have been developed using UL formulation and non-linear displacement cross-section field accounting for large rotation effects. The model examines the effect of power-law index, porosity coefficient, and skin-core-skin thickness ratios on the critical buckling temperature and post-buckling responses for various boundary conditions. The efficiency of the proposed model has been verified by shell mode.

The critical buckling temperature decreases with an increase in the power-law index p due to a higher proportion of metal in the cross-section. On the other hand, the critical buckling temperature increases with the increase of the porosity coefficient ρ because an imperfect beam is less thermally conductive than a perfect one. This relationship applies to all boundaries considered. As expected, the beam with temperature-dependent material properties shows lower thermal buckling resistance.

The proposed algorithm is more efficient and faster than numerical models that rely on shell and solid finite elements, giving it an advantage. This model can be used to design and analyse beam structures made of composite FG materials, particularly in assessing the response and loss of structural load-bearing stability in a thermal environment. In future work, the model will be expanded to include shear deformations and material plasticity. Additionally, the aim is to extend the current model to other types of composite materials, such as laminates and graphite nanotubes.

Acknowledgements

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