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Seismic Design Optimization of Concrete Cable-Stayed Bridges with "H"-Shaped Towers

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Abstract

An optimization-based strategy for designing concrete cable-stayed bridges under seismic action is presented. An initial designs module provides a convex optimization algorithm with multiple starting solutions from which local optimum solutions are obtained and the least cost solution is selected as the optimum design. The threedimensional analysis considers dead loads and road traffic live loads, geometrical nonlinearities and time-dependent effects. The modal response spectrum approach is used for seismic analysis. The design is formulated as the cost minimization subject to constraints on the displacements and stresses considering service and strength criteria defined according to the Eurocodes provisions. A constraint aggregation approach is adopted to solve the problem through the minimization of a convex scalar function obtained by an entropy-based approach. The discrete direct method is used for sensitivity analysis. The 64 design variables are the deck and towers' sizes, the cable-stays' cross-sectional areas and prestressing forces, and the towers' height. The optimization of a 312 m span bridge illustrates the features and applicability of the proposed strategy. The optimum design features a deck slenderness of 1/130 and a height of the towers (above the deck)-to-main span ratio of 0.205.

Keywords: cable-stayed bridges, optimization, seismic action, concrete, cable forces, sizing design variables, shape design variables.

1 Introduction

Cable-stayed bridges are commonly used for medium-to-long spans because of their structural and construction efficiency, as well as their economic and aesthetic advantages. Modern cable-stayed bridges present multiple inclined cable-stays providing continuous support and natural prestressing to the deck, allowing for spanning long distances with slender decks. The static and dynamic behaviour of these highly redundant structures is determined by the cable forces distribution, and the stiffness and mass of the load-bearing members (deck, towers and cable-stays). Due to their characteristic flexibility and low damping, their response to dynamic actions such as wind or earthquakes is of major relevance in the design of these structures [1, 2].

The seismic behaviour of cable-stayed bridges has been the focus of research for many authors. Abdel-Ghaffar and Nazmy [3, 4, 5] studied the three-dimensional seismic response of long-span cable-stayed bridges. Geometrical nonlinearities and uniform and multiple-support seismic excitations were considered. A step-by-step integration procedure was used to obtain the non-linear earthquake response. Relevant features of the seismic analysis are indicated, and aspects such as nonlinearities, spatial variability of the ground motion and structural configuration, affecting the seismic response are discussed. Morgenthal [6] studied the transverse seismic behaviour of cable-stayed bridges and analysed the improvement in the seismic response by using passive isolation devices in the connection between the deck and the towers. Cámara [7] investigated various analysis strategies for evaluating the linear and nonlinear dynamic behaviour of cable-stayed bridges. Furthermore, the influence of different design options and using passive devices placed in the towers to improve the bridges' seismic behaviour were also analysed. Cámara and Efthymiou [8] studied the deck-tower interaction in the transverse seismic response of cablestayed bridges. The authors considered the contribution of different vibration modes and the influence of the main span length, the tower shape, the cable-system arrangement, the width of the deck, the height of the deck above the foundation and the soil conditions. These studies highlighted the importance of the cable-suspension system layout, the deck-tower connection, and the tower shape on the seismic behaviour of these bridges.

The seismic analysis adds complexity to the challenging task of designing cablestayed bridges. This involves defining the structural system, finding the members' cross-sectional sizes, and calculating the distribution of cables' forces. The analysis should consider geometrical non-linear effects, erection stages and the timedependent behaviour of concrete. Therefore, optimization techniques are particularly suited for assisting in this design problem aiming at economic and structurally efficient solutions. A recent literature survey by the authors [9] revealed that the optimization of cable-stayed bridges is a relevant topic of research. Previous works can be grouped into two main research subjects, namely, the cables' forces optimization and the optimum design. This survey pointed out some expected future developments in this topic, which can be already identified in recent works. These include the use of metaheuristic algorithms [10, 11], artificial neural networks and surrogate models [12], the optimization of footbridges [10, 13], curved bridges [13, 14], long-span bridges and multi-span bridges with innovative cable arrangements like crossing-cables. The optimum design including the response to wind [15, 16] and earthquakes [12, 14], the reliability-based optimum design and the robust design including, for example, cable loss scenarios [17], were not amply addressed previously and represent subjects of relevance for upcoming research.

The optimum seismic design of steel cable-stayed bridges was previously studied considering modal/spectral and time-history approaches [18], the simultaneous optimization of structure and control devices [19], and curved bridges with viscous dampers as control devices [14]. Franchini et al. [12] proposed the implementation of parameterised fragility functions for the surrogate-based sensitivity analysis and performance-based optimization of cable-stayed bridges subject to seismic action. This approach was applied to the optimization of a 542 m three-span bridge with concrete towers and a steel-concrete composite deck. The cross-sectional area of the cable-stays, the towers cross-sectional dimensions, the longitudinal and transverse steel reinforcement ratios were considered as design variables. The cables prestressing forces, the cross-sectional dimensions of the deck and towers' geometry were not considered as design variables. The optimization of concrete cable-stayed bridges was previously addressed by the authors [20] but without considering shape design variables.

The main objective of this work is to present an optimization-based strategy for the seismic design of concrete cable-stayed bridges with "H"-shaped towers. This optimization problem may be non-convex and the feasible domain may be nonconnected due to the dynamic loading. The problem presents a large number of design variables of different types (shape, sizing, and mechanical), and a large number of nonlinear and conflicting constraints. A computationally costly problem with a complex design space is expected. Gradient-based and metaheuristic algorithms can be used to solve this problem, which can be posed as a single objective optimization to minimize the cost of the structure subject to a set of constraints. Here, an efficient convex optimization technique using different starting designs is proposed to solve the original non-convex problem. Concerning previous works, in this paper shape design variables are considered in concrete towers, a different solution for the deck design is adopted which implies additional design variables, and a procedure to automate the definition of initial designs was implemented.

2 Optimization strategy

The proposed strategy comprises an *Initial Solutions Module*, and an *Analysis & Optimization Module*. This strategy was implemented in a computer program developed in MATLAB environment. The corresponding flowchart is depicted in Figure 1.

The *Initial Solutions Module* efficiently provides the optimization algorithm with adequate starting designs thus contributing to the exploration of the design space. Each initial design is established by defining values for the cross-sectional dimensions of the deck and towers, as well as the height of the towers. The values of the corresponding design variables should be defined according to the usual dimensions of this type of bridge. For example, the depth of the deck should be set to achieve a deck slenderness of 1/100 to 1/120. The influence matrix method [21] is used to compute the cable-stays prestressing forces aiming to control the deck vertical displacements and the towers' horizontal displacements for the bridge under dead load. The cable-stays' cross-sectional areas are determined from the cables' axial forces under different load cases to satisfy the corresponding stress design constraints. The *Analysis & Optimization* module includes the structural analysis, the sensitivity analysis and the optimization algorithm. The finite element method was used for the three-dimensional analysis under static loading (dead load and road traffic live load) and seismic action, including geometrical nonlinearities and time-dependent effects. The deck and towers were modelled with 2-node and 12-degrees of freedom Euler-Bernoulli beam elements. To consider the second-order effects, the stiffness matrix of the beam elements includes the elastic and geometric contributions and the cables were modelled as 2-node bar elements with an equivalent modulus of elasticity given by Ernst formulation [22]. Therefore, the structural analysis was conducted iteratively to perform a second-order elastic analysis.

Figure 1: Flowchart of the optimization strategy.

Structural concrete was modelled as a linear viscoelastic material and the timedependent effects of ageing, creep and shrinkage of concrete were evaluated according to NP EN 1992-1-1 [23] formulation. For a given time interval, it is possible to calculate nodal forces equivalent to the creep and shrinkage deformations. These forces produce the same displacements field as the time-dependent effects. Thus, the stresses are calculated using only the elastic constitutive relationship between stresses and mechanical origin deformations. A previous work by the authors [24] provides detailed information about the time-dependent effects modelling.

A linear elastic behaviour of the materials (structural concrete, reinforcing steel, structural steel, prestressing steel) was adopted. The materials nonlinearities were considered in the formulation of the design constraints regarding stresses in the different structural members. Homogeneous concrete cross-sections were assumed and the steel reinforcement was considered only for design purposes.

The structural analysis under seismic action requires solving the dynamic equilibrium equation given by:

$$
\underline{M} \cdot \underline{\ddot{u}}(t) + \underline{C} \cdot \underline{\dot{u}}(t) + \underline{K} \cdot \underline{u}(t) = -\underline{M} \cdot \underline{r} \cdot \underline{\ddot{u}}_s(t)
$$
\n(1)

where $\underline{\dot{u}}(t)$, $\underline{\dot{u}}(t)$ and $\underline{u}(t)$ are, respectively, the vectors of acceleration, velocity and displacement of the structure; M , C and K are, respectively, the mass, damping and stiffness matrices of the structure; $\frac{1}{r}$ is an influence matrix relating the degrees of freedom of the structure and the ground acceleration components $\vec{u}_g^T(t) = \langle \vec{u}_g^X, \vec{u}_g^Y, \vec{u}_g^Z \rangle$ *g Y g X g* $\underline{\ddot{u}}_g^T(t) = \langle \ddot{u}_g^X, \ddot{u}_g^Y, \ddot{u}_g \rangle$ [25]. Different approaches can be adopted to solve Equation (1), such as time-history analysis or modal response spectrum analysis. In view of the computationally efficiency for an integrated analysis-and-optimization procedure, considering an elastic behaviour without control devices and a uniform support excitation, the latter approach was adopted here [18, 20]. This approach gives a set of pseudo-static forces leading to an envelope of the critical structural responses throughout the earthquakeinduced vibration process. The natural vibration frequencies and the corresponding mode shapes are obtained from the eigenvalue and eigenvector problem

$$
[\underline{K} - \lambda \cdot \underline{M}] \cdot \underline{\phi} = 0 \tag{2}
$$

where $\lambda = \omega^2$ are the eigenvalues or characteristic values representing the square of the free vibration frequencies (ω) and ϕ represents the eigenvectors or mode shapes of the vibrating system [25]. This problem was solved using the MATLAB function *eigs*. A lumped mass matrix formulation was considered for the beam and bar elements used in the bridge model. The structure stiffness matrix, including elastic and geometric contributions, was evaluated in the dead-load permanent state [5, 26].

The seismic action was quantified using the Eurocode 8 [27] elastic response spectra. The maximum spectral accelerations, $S_{ai}(\xi_i, T_i)$, are obtained for a given damping ratio, ξ ^{*i*}, and vibration period, T ^{*i*}, of each mode and for each direction. A constant damping ratio of $\xi = 3\%$ was considered [5, 6, 28]. Considering the modal coupling that is present in the dynamic response of these bridges, the complete quadratic combination (CQC) was used for the combination of the maximum contribution of each mode [29]. As indicated in NP EN 1998-2 (2023) [30], the square root of the sum of squares (SRSS) rule was used to combine the seismic effects in the longitudinal, transverse, and vertical directions.

The design of concrete cable-stayed bridges under seismic action seeks to minimize the bridge cost while satisfying a large number of design constraints concerning service and strength criteria. Finding the active set of constraints in gradient-based nonlinear programming algorithms may pose a considerable difficulty in solving problems with a large number of constraints. Constraint aggregation approaches are particularly suited for solving problems with a large number of constraints using gradient-based algorithms. Classical (least p-norm and Kreisselmeier-Steinhauser (KS) function) or induced aggregation (induced exponential and induced power aggregation) methods can be adopted. Therefore, the problem can be solved as a single objective optimization problem to minimize the cost subject to aggregated constraints or as a multi-objective optimization by aggregating the cost and the design goals, defined by the constraints, in a single objective function. As in previous works by the authors, concerning the optimization of cable-supported bridges, the latter approach was adopted here. The multi-objective problem is solved by the minimization of a convex scalar function (Equation (3)) obtained through an entropy-based approach [31]. The scalar function aggregates all the design goals and creates a convex approximation close to the boundaries of the original non-convex domain. This function includes all the constraints with different probabilities of becoming active. As iterations proceed, there is decreasing uncertainty about which constraints are more relevant to find the optimum. This procedure reduces the cost objective with respect to previous iterations and simultaneously keeps all the constraints within limits. The design constraints, $g_i(x)$, do not have an explicit algebraic form, thus, the problem is solved using an explicit approximation given by the Taylor series expansion of all the constraints, around the current design variable vector, truncated after the linear term

$$
\min F(\underline{x}) = \min \frac{1}{\rho} \ln \left[\sum_{j=1}^{M} e^{-\rho \left(g_j(\underline{x}) + \sum_{i=1}^{N} \frac{dg_j(\underline{x})}{dx_i} dx_i \right)} \right]
$$
(3)

where *x* is the vector of design variables, *M* is the number of design goals, *N* is the number of design variables, $g_i(x)$ is the *j-th* design goal, $dg_i(x)/dx_i$ is the sensitivity of the *j-th* design goal with respect to *i-th* design variable. The aggregation parameter *ρ* must not be decreased during the optimization process and its value should be tuned for each problem. However, similar results are obtained with values ranging from 100 to 2000. The accuracy of the explicit approximation was ensured by using bound constraints with move limits. The MATLAB function *fmincon*, which minimizes a scalar function of several variables subject to bound constraints using a sequence of quadratic problems, was selected to minimize the objective function. Different types of design variables were considered, namely, sizing, mechanical and shape design variables. Sizing variables refer to the cross-sectional dimensions of deck, towers and cable-stays. These variables directly influence the mass, stiffness and cost of the structure. A second type refers to the mechanical design variables corresponding to the cable-stays' prestressing forces. These variables do not directly influence the cost but are fundamental in controlling the structural response of cable-stayed bridges. A third type was considered to characterize the geometry of the towers. These design variables affect the cable-supporting system, the mass, stiffness, and cost. The longitudinal and transverse steel reinforcement areas were considered constant design parameters with usual practical values defined as percentages of the concrete crosssectional area. Figure 2 shows the 64 design variables considered.

Figure 2: Design variables, material properties and unit costs.

In the constraint aggregation approach adopted, the cost is considered the first design goal and can be expressed as

$$
g_1(\underline{x}) = \frac{C}{C_0} - 1 \le 0
$$
\n
$$
\tag{4}
$$

where C is the current cost of the structure and C_0 represents the initial cost of each analysis and optimization cycle. This approach always prioritises cost as one of the optimization objectives. A second set of objectives aims to limit the vertical displacements of the deck and the horizontal displacements of the towers under service conditions and considering the time-dependent effects

$$
g_2(\underline{x}) = \frac{|\delta|}{\delta_0} - 1 \le 0
$$
\n⁽⁵⁾

where δ and δ ⁰ are the displacement value and the limit value for the displacement under control, respectively. For the long-term analysis of the bridge, values of *L/*1000 and *H/*1000 were considered for the limits on the vertical and horizontal

displacements, respectively. *L* and *H* represent the main span length, and the height of the towers, respectively.

The stress constraints concerning the deck and towers' members were defined based on the NP EN 1992-1-1 [23] and NP EN 1993-1-1 [32] provisions. Generally, these constraints can be expressed as

$$
g_3(\underline{x}) = \frac{\sigma}{\sigma_{\text{allow}}}-1 \le 0
$$
\n(6)

where σ and σ_{allow} are the acting stress and the corresponding allowable stress, respectively. Different values for the allowable stress were considered for service conditions and strength verification of concrete members. For service conditions, 4.1 MPa and 22.5 MPa were used for tension and compression, respectively. For strength verification, the allowable value represents the structural concrete member's resistance, including reinforcement, evaluated according to the acting internal forces, such as bending and axial force, biaxial bending and axial force, or shear force. For structural steel members (deck transverse beams) under service conditions, the acting stress is evaluated according to the Von Mises criteria, and the allowable stress corresponds to the nominal value of the steel yield strength. For strength verification, the acting stress represents the acting internal force (bending moment or shear force) and the allowable value is the corresponding resistance evaluated assuming Class 1 or Class 2 cross-sections. Another set of constraints concerns the cable-stays' stresses and can be written as

$$
g_4(\underline{x}) = \frac{\sigma}{k \cdot f_{pk}} - 1 \le 0
$$
\n⁽⁷⁾

$$
g_5(\underline{x}) = 1 - \frac{\sigma}{0.10 \cdot f_{pk}} \le 0
$$
\n⁽⁸⁾

where σ and f_{pk} are the acting stress and the characteristic value of the prestressing steel tensile strength, respectively. The value of *k* in Equation (7) was considered equal to 0.50 for service conditions and 0.74 for strength verification. Equation (8) refers to a lower limit for tension in the cable-stays to ensure their structural efficiency.

The optimization algorithm requires the gradients of the objective function and all the design constraints with respect to the design variables. This information is provided by the sensitivity analysis. From the available approaches, the discrete direct method was chosen, using both analytical and semi-analytical derivatives.

3 Numerical example

The numerical examples concern the optimization of a symmetrical concrete cablestayed bridge with a total length of 312 m and a main span of 160 m (Figure 3). The total height of the towers is given by $x_{63}+x_{64}$, with the deck placed 20 m above the foundation. A semi-harp cable arrangement with lateral suspension and a total of 80 cables was considered. The cable spacing is 8 m on the deck and is defined by variable

*x*⁶⁴ on the towers. A full suspension solution was chosen as usual in the design of cable-stayed bridges in earthquake-prone areas. The deck is simply supported at the abutments, continuously supported by the inclined cable-stays along the bridge length and it is not supported at the towers. The deck-tower connection is only in the transverse direction and was modelled with link elements with stiffness of 1.0×10^3 kN/m. A beam-and-slab cross-section was considered for the deck. The slab is supported by longitudinal concrete beams and transverse steel beams. The use of steel transverse beams allows reducing the deck weight, which favours both the static and seismic behaviour. A similar solution was adopted in the Vasco da Gama bridge, in Lisbon, Portugal. The transverse beams feature an "I"-shaped profile and 4 m spacing. "H"-shaped towers with rectangular hollow sections were considered. Given that the soil-structure interaction was disregarded, fixed supports were adopted for the towers. Beam elements were used to model the towers and the deck's longitudinal and transverse beams. Axially stiff bar elements were used in modelling the deck as an inplane rigid diaphragm. The bridge finite element model is depicted in Figure 3 and has a total of 218 nodes and 533 finite elements. The properties and unit costs of the materials considered are presented in Figure 2.

Figure 3: Finite element mesh of the bridge example.

Six load cases were defined to check the relevant service and strength design constraints. The first case refers to the bridge under dead load (self-weight and an additional dead load of 2.5 kN/m^2 corresponding to flooring, walkways, safety barriers and guardrails) at the end of construction. The second case corresponds to the long-term analysis (18,250 days) of the bridge under the quasi-permanent load combination (dead load plus 20% of road traffic live load). To consider the most unfavourable effects of the road traffic live load (5 kN/m^2) , three additional load cases were considered. These correspond to the live load placed on the entire deck length, or only in central or side spans. The sixth load case corresponds to the bridge under dead load, plus 20% of the live load on the entire deck and the seismic action. The construction stages are relevant in the design of cable-stayed bridges. However, the current paper focuses in the static and seismic response of the complete bridge and thus, the erection stages were not directly considered. The response spectrum used to quantify the seismic action was defined from the Eurocode 8 [27] elastic response spectra considering a type A ground (rocky soil) and type 1 spectrum (more dangerous because presents higher spectral accelerations for the long period area that characterizes these structures). The value of the design ground acceleration, $a_{g} = 0.5 \cdot g$, was defined according to the Portuguese National Annex for a seismic zone 3 and an importance class III. Considering the same document the design ground acceleration in the vertical direction was taken as $a_{vg} = 0.75 \cdot a_{g}$. Cable-stayed bridges present long vibration periods, thus, the requirements about spectra for periods longer than 4 s were considered. A behaviour factor, $q = 1.0$, was adopted [30]. Considering the type of ground and the distance between supports the spatial variability of the seismic action was disregarded.

The problem features 64 design variables and more than 2300 design constraints for the six load cases. To explore the design space, eight initial designs were considered and optimized. These were defined by varying the deck sizes and the height of the towers aiming at deck slenderness between 1/100 and 1/130, and a height of the towers (above the deck)-to-main span ratio between 0.20 and 0.275. The presented results refer only to the initial and final values of the optimum solution. Figure 4 presents the evolution of the bridge cost throughout the optimization process. Figure 5 depicts the initial and final values of the cables' cross-sectional areas and prestressing forces for the optimum solution. The cables are numbered from the abutments to the central span.

Figure 4: Bridge cost *vs.* number of iterations – optimum solution.

The optimum solution presents a cost reduction of 14.5% concerning the initial solution. This is due to a reduction in the sizing design variables of the deck and towers (Table 1). In the optimum solution the deck, towers and cable-stays represent 31.4%, 9.5% and 59.1% of the total cost, respectively. The value obtained for the cable-stays in mainly due to a fixed cost (18.500 ϵ /cable). The cost of the deck does not include the slab and refers only to the longitudinal and transverse beams. The active constraints at the optimum are the deck vertical displacements for load case 2, the cables stress for load cases 2 to 5, the deck transverse beams bending resistance for load cases 3 to 5, the deck longitudinal beams bending resistance for load case 5 and the towers biaxial bending and axial force resistance for load cases 4 to 6. For the optimum solution the first three vibration periods are 3.73 s, 3.14 s and 2.76 s, corresponding to transverse, vertical and longitudinal modes, respectively. The vibration period of the $30th$ mode is 0.51 s. Of the 1276 modes that could be included in the modal superposition, only the first 30 modes were considered. For the optimum solution, these 30 modes represent 90.7, 76.7 and 78.2% of mass participation in the longitudinal, transverse and vertical directions, respectively.

Table 1: Initial and final values of the cost and design variables 41 to 64 – optimum solution.

Figure 5: Initial and final values of the cable-stays' prestressing forces and cross sectional areas – optimum solution.

4 Conclusions and Contributions

The following conclusions can be drawn:

- The seismic design of concrete cable-stayed bridges with "H"-shaped towers is solved as an optimization problem to minimize the cost subject to constraints on the displacements and stresses considering service and strength criteria.
- A convex optimization strategy is used to solve the original non-convex problem. This strategy combines a constraint aggregation approach to efficiently solve a problem with a large number of design variables through a gradient-based optimization, and a procedure to automate the definition of initial designs. Local optimum solutions are obtained from each starting design and the least cost solution is selected as the optimum design.
- By rearranging the stiffness and mass distribution between the load-bearing members, the algorithm finds minimum-cost solutions while improving structural response under both static loading and seismic action. The optimum solution satisfies all the design constraints and features cost reduction due to a decrease in the values of the sizing design variables of the deck and towers.
- In the optimum solution the deck, towers and cable-stays represent 31.4%, 9.5%, and 59.1% of the total cost, respectively. The design is governed by the deck's vertical displacements, the cables' stress, and the deck and towers' resistance.
- Regarding the tower geometry, the optimum solution features a height of the towers (above the deck)-to-main span ratio of 0.205. The solution adopted for the deck allows weight reduction which favours static and seismic performance. A deck slenderness of 1/130 is obtained in the optimum solution.
- Future developments should consider towers with different typologies, such as, "A"-shaped and "inverted Y"-shaped. Different intensities of the seismic action should be also considered. Different support conditions for the deck should be investigated. These include the deck-tower connection in the longitudinal and transverse directions and the bearings at the abutments. The simultaneous optimization of structure and vibration control devices should be also considered.
- Combining the gradient-based algorithm with a global search procedure will be addressed in upcoming research, thus, contributing to the exploration of the complex design space.

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