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Optimal Design of Lattice Domes by Means of a Constrained Force Density Method

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Abstract

In this contribution the design of reticulated domes is dealt with, exploring the optimal solutions that can be retrieved by a form-finding approach. To this goal a numerical tool is implemented to address the design of reticulated shells through funicular analysis. The force density method is implemented to cope with the equilibrium of reticulated shells whose branches are required to behave as bars. Optimal networks are sought by coupling the force density method with techniques of sequential convex programming that were originally conceived to handle formulations of size optimization for elastic structures. The Maxwell number, which is the sum of the force-times-length products for all the branches in the spatial network, is used as objective function to be minimized, whereas constraints on the length of the branches are enforced. Funicular networks that are fully feasible with respect to the set of local enforcements are retrieved in a limited number of iterations, with no need to initialize the procedure with a feasible starting guess. Optimal solutions are explored, considering different types of grids, i.e. different types of lattices, while considering self-weight.

Keywords: form-finding, structural optimization, force density method, mathematical programming, lattice domes, lightweight structures.

1 Introduction

Lattice shells have double curvature, while consisting of branches that mainly undergo axial forces [1, 2]. Optimal shapes for such kind of structures can be conveniently investigated by means or equilibrium-based methods, e.g. funicular analysis, see e.g. [3, 4, 5]. Reticulated shells can be modelled as statically indeterminate networks of vertices and edges of prescribed topology. Boundary supports are given at the restrained nodes of the network; unrestrained ones are in equilibrium with the applied point loads. Introducing the force densities, i.e. the ratio of force to length in each branch of the network [6], the equations governing the equilibrium of the unrestrained nodes become linear and uncoupled in the three spatial directions.

In this contribution a generalization of the approach presented in [7] is explored, by embedding the Force Density Method (FDM) within a multi-constrained minimization problem. Due to its peculiar form, this problem can be efficiently solved through techniques of sequential convex programming [8] that were originally conceived to handle large-scale multi-constrained formulations of size optimization for elastic structures, see [9] among the others.

The research of the optimal shape of reticulated shells is made by adopting an alternative objective function, i.e. the Maxwell number, which is the sum of the force-times-length products for all the edges of the spatial network. According to Maxwell's theorem [10], this quantity is the same as the sum of the load-times-distance products for all the forces acting upon the network (the distance is from an arbitrary origin to the point of application of the load), see also discussions and examples in [11, 12]. Constraints are of geometric type, being related to the minimum and maximum length of the members and to the allowed range for the height of the nodes of the spatial network.

In the next sections, a brief overview of the force density method (FDM) is given, and the multi-constrained problem is presented. A numerical example is shown to demonstrate the method and draw some preliminary conclusions.

2 Force density method

The "force density method" [6] is used to handle the equilibrium of spatial networks. A funicular network consists of $n_s = n + n_f$ nodes and m branches, which play as bars. The axes of the Cartesian reference system with origin O are labelled as x, y, and z. Hence, $\mathbf{x}_s$, $\mathbf{y}_s$, $\mathbf{z}_s$ are vectors gathering the coordinates of the n_s nodes: $\mathbf{x}$, $\mathbf{y}$, $\mathbf{z}$ refer to the n unrestrained nodes, i.e. the nodes subject to external forces; $\mathbf{x}_f$, $\mathbf{y}_f$, $\mathbf{z}_f$ collect the n_f restrained nodes, i.e. those where reactions arise. The connectivity matrix that fully describes the shape of the grid is $\mathbf{C}_s$, having subset $\mathbf{C}$ for the unrestrained nodes and $\mathbf{C}_f$ for the restrained ones. The vectors that collect the coordinate difference of the nodes along the axis x, y, z are denoted by $\mathbf{u}$, $\mathbf{v}$, $\mathbf{w}$, respectively:

$$\mathbf{u} = \mathbf{C}_s\mathbf{x}_s, \quad \mathbf{v} = \mathbf{C}_s\mathbf{y}_s, \quad \mathbf{w} = \mathbf{C}_s\mathbf{z}_s. \tag{1}$$

The force densities, i.e. the ratios force to length for each branch of the network, are stored in $\mathbf{q} = \mathbf{L}^{-1}\mathbf{s}$, being $\mathbf{s}$ the vector that collects the forces in the m branches. The length of the branches $l_i = \sqrt{u_i^2 + v_i^2 + w_i^2}$ is gathered in the square matrix $\mathbf{L} = \text{diag}(\mathbf{l})$. Only gravity loads are considered in this study, especially self-weight. Vertical point forces are prescribed at the unrestrained nodes through vector $\mathbf{p}_z$. Due to the introduction of the vector $\mathbf{q}$, the equilibrium of the unrestrained nodes is given by a set of linear equations that are uncoupled in the three axes, i.e.:

$$\begin{aligned}
&\mathbf{C}^T\mathbf{QCx} + \mathbf{C}^T\mathbf{QC}_f\mathbf{x}_f = \mathbf{0}, \\
&\mathbf{C}^T\mathbf{QCy} + \mathbf{C}^T\mathbf{QC}_f\mathbf{y}_f = \mathbf{0}, \\
&\mathbf{C}^T\mathbf{QCz} + \mathbf{C}^T\mathbf{QC}_f\mathbf{z}_f = \mathbf{p}_z,
\end{aligned} \tag{2}$$

being $\mathbf{Q} = \text{diag}(\mathbf{q})$.

3 Optimization problem

A multi-constrained minimization is stated in terms of any set of force densities $\mathbf{q}$:

$$\left\{\begin{array}{lll}
\min\limits_{q_i \le 0} \; f & & \text{(3a)} \\
\text{s.t.} & \mathbf{C}_x^T\mathbf{QC}_x\mathbf{x} + \mathbf{C}_x^T\mathbf{QC}_{fx}\mathbf{x}_f = \mathbf{0}, & \\
& \mathbf{C}_y^T\mathbf{QC}_y\mathbf{y} + \mathbf{C}_y^T\mathbf{QC}_{fy}\mathbf{y}_f = \mathbf{0}, & \text{(3b)} \\
& \mathbf{C}_z^T\mathbf{QC}_z\mathbf{z} + \mathbf{C}_z^T\mathbf{QC}_{fz}\mathbf{z}_f = \mathbf{p}_z, & \\
& \left(\dfrac{l_i}{l_{min}}\right)^2 \ge 1 \quad \text{for } i = 1...m, & \text{(3c)} \\
& \left(\dfrac{l_i}{l_{max}}\right)^2 \le 1 \quad \text{for } i = 1...m, & \text{(3d)} \\
& z_j(\mathbf{q}) \ge z_j^{min} \quad \text{for } j = 1...n, & \text{(3e)} \\
& z_j(\mathbf{q}) \le z_j^{max} \quad \text{for } j = 1...n, & \text{(3f)}
\end{array}\right.$$

In the above statement, the objective function accounts for the sum of the force-times-length products computed in each branch of the network [10]. Since anti-funicular networks are dealt with, see side constraints in Eqns. (3), one has:

$$f = -\mathbf{s}^T\mathbf{l} = -\mathbf{q}^T\mathbf{Ll}. \tag{4}$$

The system of Eqn. (3b) states the equilibrium of the unrestrained nodes in the three spatial directions, to compute $\mathbf{z}$ from $\mathbf{q}$.

Eqns. (3c) and (3d) are used to prescribe the minimum (l_{max}) and maximum (l_{max}) value of the length of each branch of the network. The coordinate difference of the connected points given in Eqn. (1) are used to enforce these geometric constraints in a straightforward way.

Eqns. (3e–3f) are two sets of inequalities that prescribe lower and upper limits for the nodal coordinates $\mathbf{z}$. The design domain is such that each one of the n coordinates z_j must be bounded from below by z_j^{min} and from above by z_j^{max}.

The multi-constrained minimization problem is solved by means of the Method of Moving Asymptotes [10], see the discussion in Section 1. Being MMA a first order approach, the sensitivity of the objective function and constraints with respect to the minimization variables in $\mathbf{q}$ is needed, see e.g. in [7].

4 Numerical example

The procedure sketched above is applied to the optimal design of lattice domes, see e.g. [13].

The topology of a so-called Schwedler dome, see e.g. [14], is adopted to test the algorithm. Such a structural topology includes intersecting ribs, rings, and diagonal elements, see Figure 1(a). The dome has a radius equal to $r = 6\,\mathrm{m}$. The design domain is such that the lower and the upper bound of the vertical coordinates read $z_{min} = 3.00\,\mathrm{m}$ and $z_{max} = 8.00\,\mathrm{m}$, respectively. It is considered that the minimum and the maximum length allowed to the members are $l_{min} = 0.75\,\mathrm{m}$ and $l_{max} = 2.00\,\mathrm{m}$, respectively.

The goal is to investigate the shape of the dome, finding the layout that minimizes the Maxwell number in Eqn. (4), such that constraints on the length of the bars and on the location of the vertical coordinates of the nodes are met. Self-weight is addressed, considering a cross-section with area equal to $0.3 \times 0.3\,\mathrm{m}^2$ and a weight per unit length that is $25\,\mathrm{kN/m}^3$.

It must be remarked that the system in Eqn. (2) remains linear if the load vector is design-independent. Hence, an iterative procedure is implemented, by updating the load vector after each optimization run. Figure 1 refers to the initial step, whereas Figure 2 is related to the last one. The latter, herein the 5-th one, is characterized by a relative variation in the objective function (with respect to the previous iteration) that is less than 0.01.

Concerning the first analysis, Figure 1(a) reports the nodal loads, whereas Figure 1(b) represents the initial guess, i.e. the layout found enforcing the same force density ($25\,\mathrm{kN/m}$) for all the branches. As one may see, geometric requirement are not met, whereas a network that is quite different with respect to the initial scheme arises. All the members are active (having non-zero force densities). Figure 1(c) presents the optimal solution, that is fully feasible with local constraints and is a major variation with respect to the layout of the initial guess. Indeed, only the original intersecting ribs and diagonal elements remain, whereas rings are inactive (see the force intensity map represented in the relevant picture). Reference is made to [15] for an insight of the structural rigidity of such kind of pin-jointed space trusses with cyclic symmetry.

The result found in final step is that of Figure 2(b), which is a further modification of the initial result.

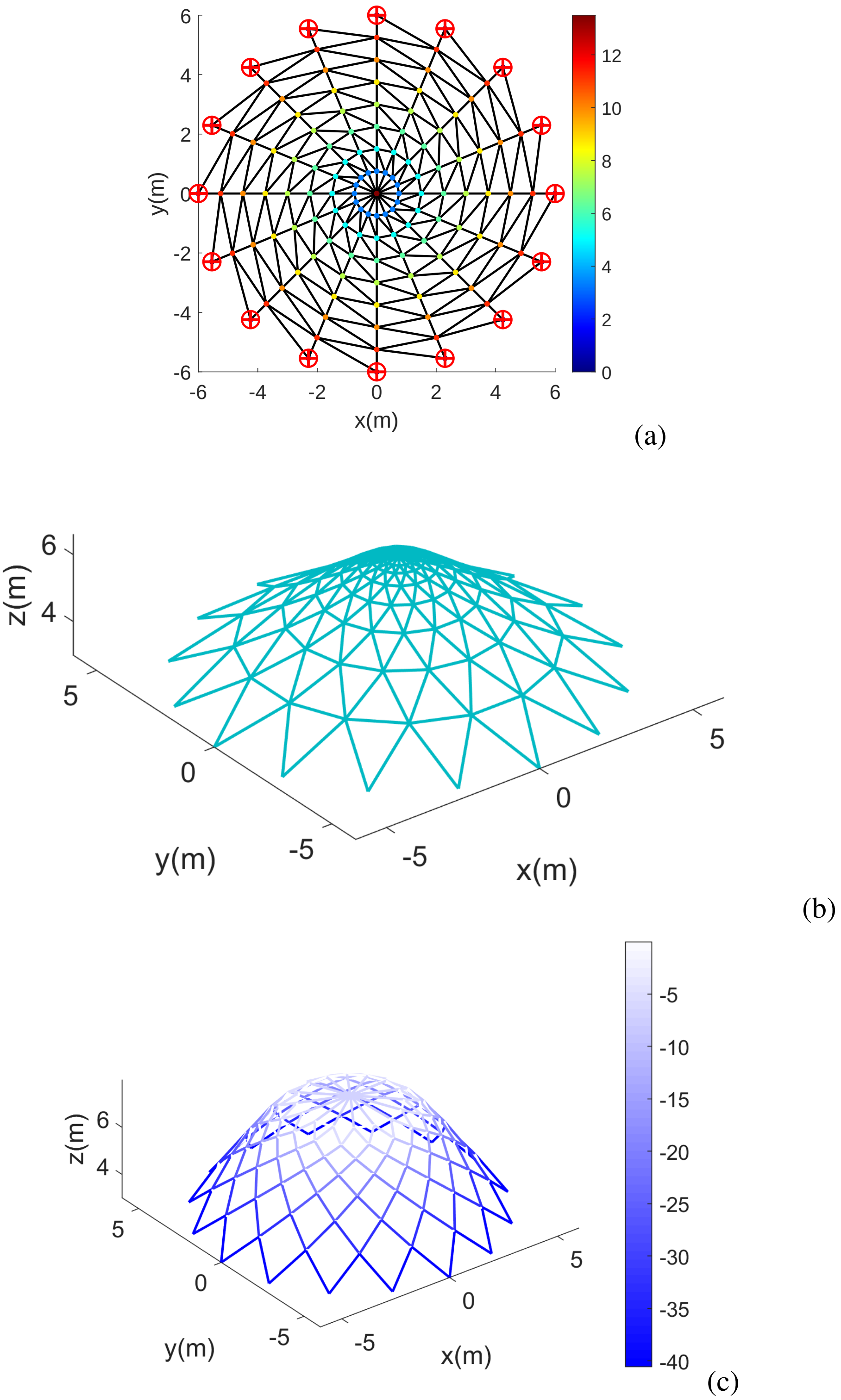

Figure 1: Initial run. Topology of the grid and nodal loads in kN (a), starting guess (b), optimal network and element forces, in kN (c). Objective function at convergence $6643\,\mathrm{kNm}$.

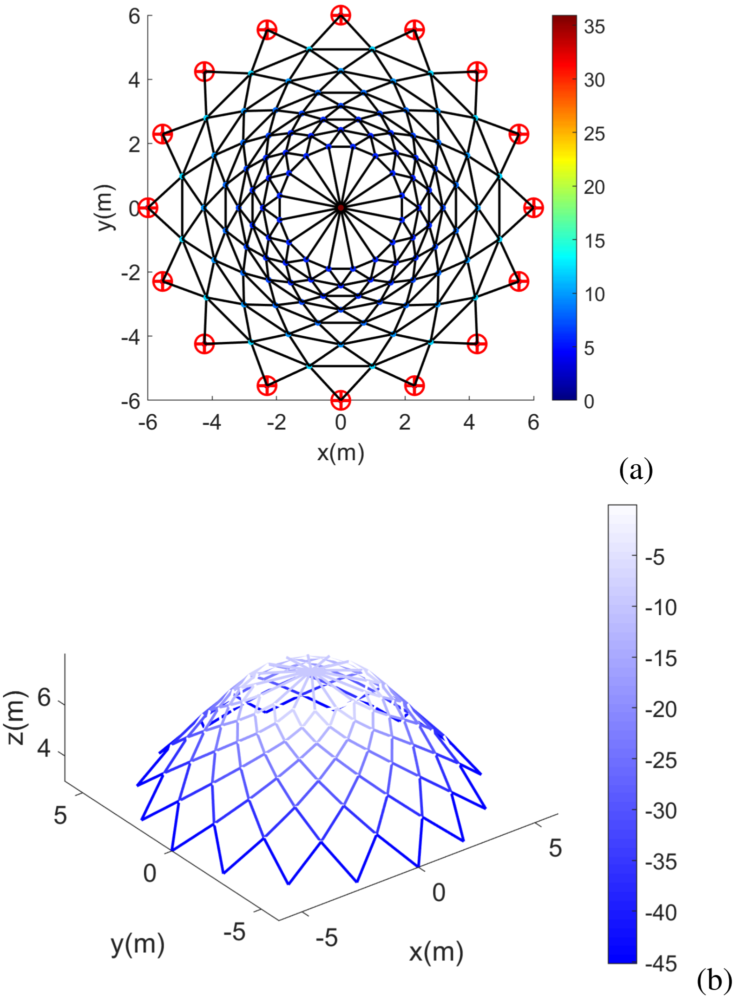

Figure 2: Final run. Topology of the grid and nodal loads in kN (a), optimal network and element forces, in kN (b). Objective function at convergence 7576 kNm.

5 Concluding remarks

In this contribution, a numerical tool has been implemented to address the design of reticulated domes through funicular analysis. As investigated in the recent literature, the force density method can be conveniently implemented to cope with the equilibrium of spatial networks of trusses, especially when coupling the form-finding tool with optimization routines. The Maxwell number, which is the sum of the force-times-length products for all the branches in the spatial network, has been used as objective function. Constraints have been enforced on the coordinates of the nodes, to prescribe a feasible design domain, and on the geometry of the members, to control their minimum and maximum length. The arising multi-constrained problem has been handled by techniques of sequential convex programming, exploiting the particular form of the equations that recalls that originally found in problems of size optimization.

A numerical example has been shown retrieving networks for dome-like reticulated shells that are fully feasible with respect to the enforced local constraints. In particular, an iterative procedure has been implemented to cope with self-weight without affecting linearity of the equations of the force density method.

The ongoing research is focused on testing the proposed procedure for different initial geometries, inspired by [14], in order to discuss optimal solutions (in terms of optimality with respect to the adopted measure, i.e. the Maxwell number). Further research includes an extension of the procedure to account for probabilistic loading, see [16].

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