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# Reliability-Based Optimization of Steel Beam Designs for Elevated Temperature Applications

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## Abstract

In this article, a novel algorithm is presented for reliability-based topology optimization of steel beams under elevated temperatures. The proposed framework integrates elastoplastic limit analysis by employing the concept of a plastic ultimate load multiplier. Additionally, the algorithm incorporates the location of the applied load as a random variable to enhance reliability-based design. This technique facilitates the analysis of other critical characteristics, such as geometrical imperfections, volume fraction, and material properties, all assumed to follow a normal distribution to address uncertainties. The optimal layouts are generated using these improvements within the bi-directional evolutionary structural optimization method to minimize structural weight while maintaining high performance. Furthermore, different reliability indices result in varied topologies, indicating the sensitivity of the optimization process to these values. By incorporating the reliability index as a constraint, the algorithm effectively regulates the optimization procedure. This method offers an efficient design strategy that considers probabilistic conditions, thereby enhancing the safety and durability of structures exposed to high temperatures.

**Keywords:** thermoelastic-plastic, reliability-based design, topology optimization, BESO, steel beams.

# 1 Introduction

Over the past few years, steel has gained significant popularity as a preferred option for a wide range of construction projects, thanks to its exceptional strength and durability. Nevertheless, the behavior of structural steel can be greatly influenced by high temperatures, resulting in unfavourable alterations to its material characteristics. These modifications result in decreased strength, stiffness, and ductility, which present obstacles for maintaining structural integrity[1,2]. Quiel and Garlock[3] investigated the structural response of steel high-rise building fire by incorporating a finite element (FE) model.

By adopting the elasto-plastic model, it is possible to enhance the resilience of structural elements when exposed to high temperatures. This model takes into consideration the non-linear characteristics of materials when subjected to loads, especially in the plastic range[4].

Furthermore, methods for refining structural designs for high-temperature conditions have been developed through topology optimization. These methods determine the most effective material distribution, resulting in optimized structural performance[5]. For instance, Kambampati[6] developed a formulation for topology optimization that takes into account both mechanical and thermal loads. The main objective was to minimize stress and compliance while adhering to temperature constraints.

As the pursuit of optimal structural designs via topology optimization progresses, the incorporation of reliability becomes an increasingly vital aspect in enhancing the optimization procedure. By placing importance on the mitigation of uncertainties pertaining to materials, geometries, and other crucial elements, reliability-based design represents a significant progression in guaranteeing the durability and safety of structures[7]. Habashneh and Rad [8] presented a computational methodology that enables the optimization of the topology of thermoelastic structures using reliability-based design. Furthermore, an optimization approach was introduced by Bruggi et al.[9]. This approach integrated homogenization-based topology optimization and accounted for load uncertainty.

Drawing upon our prior investigations, the present research aims to enhance the efficiency of structure design in high-temperature environments through the utilization of the bi-directional evolutionary structural optimization (BESO) algorithm integrated within the elastoplastic limit analysis framework. The aforementioned research work established the groundwork for our present investigation, which further develops it by incorporating imperfections in nonlinear analyses into the reliability-based design. Due to their critical relationships with structural performance and safety at elevated temperatures, the position of the applied load, geometrical imperfections, and material volume are considered randomly.

Moreover, the proposed research investigates the effects of the optimized topology that results in the distribution of temperature within the structures, thereby contributing to the progression of knowledge regarding their thermal capabilities.

The prospective enhancement of probabilistic optimization designs for elastoplastic structures under high temperature conditions will be underscored in the subsequent sections.

## 2 Methods

This section introduces the reliability-based optimization problem for steel beams under elevated temperatures, taking into account geometric nonlinearity and elastoplastic material behavior. The optimization objective is to minimize structural weight while adhering to constraints related to reliability-based design, thermoelastoplastic behavior, and the desired volume of the beam at the end of the optimization process. Thus, the equations for the proposed optimization method can be formulated as follows:

$$\text{Minimize: } C = u^T Ku \quad (1.a)$$

$$\text{Subject to: } \frac{V^*}{V_0} - V_f \leq 0 \quad (1.b)$$

$$x_i \in \{0,1\} \quad (1.c)$$

$$\lambda_j \geq \underline{\lambda} > 0 \quad (1.d)$$

$$m_s - m_p \leq 0 \quad (1.e)$$

$$Ku = f \quad (1.f)$$

$$\beta_{target} - \beta_{calc} \leq 0 \quad (1.g)$$

The value of  $K$  represents the global stiffness matrix, while  $C$  represents the mean compliance and  $u$  represents the displacement vectors. Furthermore,  $V^*$  signifies the structural total volume. In addition,  $V_0$  denotes the design domain's volume, whereas  $V_f$  denotes the volume fraction. Furthermore, the binary variable of design  $x_i$  is capable of assuming two values: zero (1) to signify the presence of an element, and one (0) to denote its absence. Notably, the vector of loading, which is the result of the combine thermomechanical loading  $f_{heat}$  and mechanical loading  $f_m$ , denoted as  $f = f_m + f_h$ . Equation (1. d) illustrates the constraint on buckling load factors, where  $\lambda_j$  represents the  $j$ th buckling load factor associated with the provided load cases and  $\underline{\lambda}$  represents the minimum buckling load factor value. The plastic limit load multiplier is additional constrain introduced by Equation (1. e). An admissible load multiplier, represented by  $m_s$ , must be equal to or less than the plastic load multiplier ( $m_p$ ), according to the principle of statics. A nonlinear thermal elastoplastic analysis determined the value of the displacement parameter  $u$  in Equation (1. a). The

termination of the optimization process occurs when the structure undergoes complete plastic collapse, as enforced by Equation (1.e). Relevant parameters, including geometrical imperfections,  $V_f$ , material properties, and applied load location, are considered random variables, as stated previously. It is important to mention that the Gaussian distribution model is employed, due to its straightforwardness, which enables the calculation of the entire distribution using only two parameters: the mean and standard deviation. Also, The Monte Carlo (MC) technique is used to calculate the reliability indices. Following this, Equation(1.g) represents the reliability constraint related to the volume fraction in which the mathematical form of the reliability index ( $\beta$ ) developed as:

$$\beta_{target} - \beta_{calc} \leq 0 \quad (2)$$

To calculate  $\beta_{target}$  and  $\beta_{calc}$ :

$$\beta_{target} = -\Phi^{-1}(P_{f,target}); \quad (3)$$

$$\beta_{calc} = -\Phi^{-1}(P_{f,calc}). \quad (4)$$

### 3 Results

The BESO algorithm is used to address the reliability-based topology optimization problem of steel I-beam. This problem takes into account uncertain load positions, incorporates thermoelastic-plastic analysis, and considers the influence of initial geometric imperfections. The effectiveness and robustness of the proposed approach are demonstrated by examining a steel I-beam. BESO parameters under consideration are as follows: the convergence criteria ( $\tau$ ) is 0.1%, the filtering radius ( $r_{min}$ ) is assumed to be 138 mm, evolutionary ratio (ER) is equal to (1%). In order to achieve the desired material distribution, it is important to designate a target volume fraction of 50%. In this research, a finite element model was developed and verified per the methodology described by Habashneh et al.[10] is utilized. The introduction of imperfection occurs through the consideration of the initial two linear buckling modes to account for the effects of global buckling. By assuming  $l$  is the length of the beam, it is commonly accepted that the beam's initial geometric imperfection equals  $l/1000$ . The steel I-beam under consideration is illustrated in Figure 1. MC simulation number is assumed to be  $1 \times 10^9$ . Moreover, Figure 2 showcases the buckling modes that have been considered for subsequent analysis. The determination of the modes involves performing linear buckling analysis at the beginning, which enables the identification of the critical buckling behavior of the structure. It is important to mention that factors related to proportional limit, yield stress, and elastic modulus, are reduction factors at elevated temperature which is determined using reference [11].

The verification of the finite element model's accuracy is accomplished through a comparison between the load-deflection diagram generated in this study and the findings documented by Habashneh et al.[10]. As shown in Figure 3, the analysis demonstrates a significant level of acceptance.

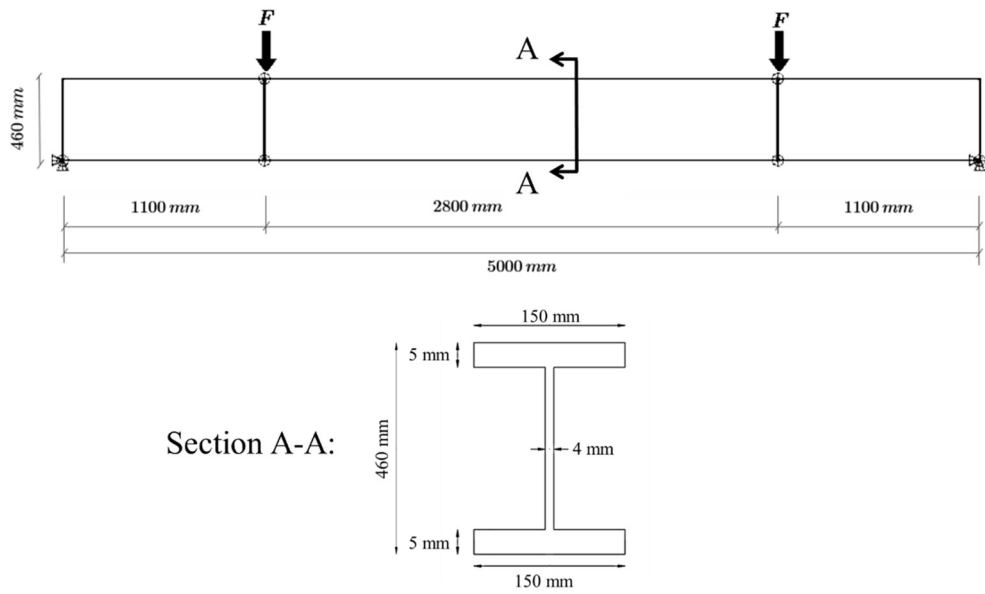


Figure 1. Considered Steel I-beam

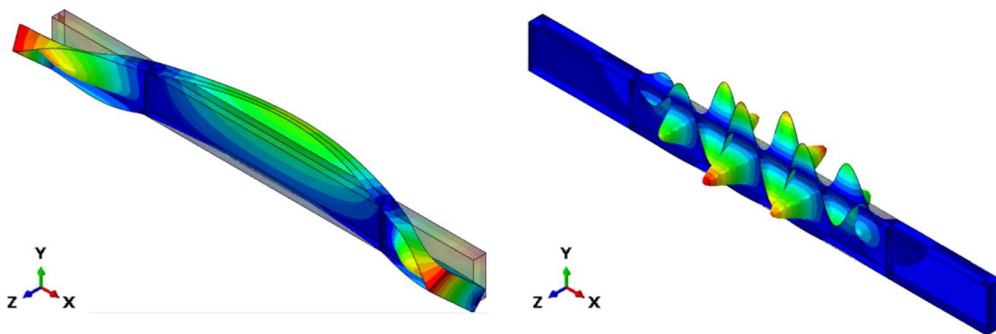


Figure 2. First and second buckling modes

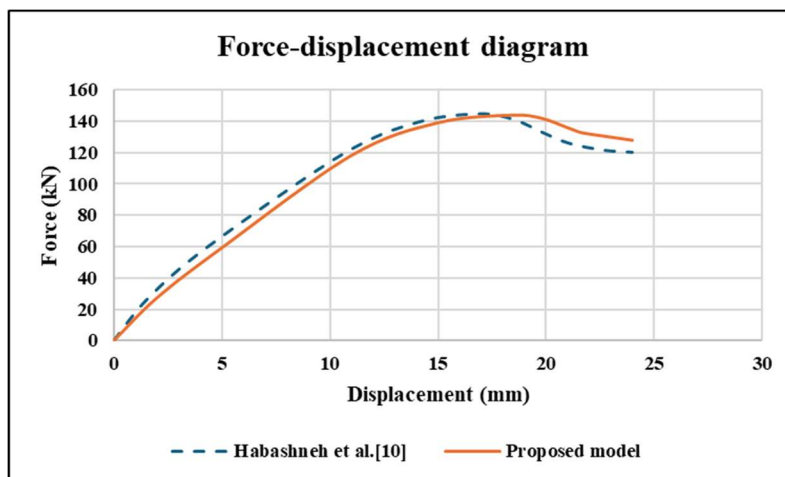


Figure 3. Applied force and corresponding displacement curve

By taking into account various values of  $\beta_{target}$ , Table 1 presents the outcomes of optimizing the topology of steel I-beam under elevated temperature (800°C) using reliability-based design. The results suggest that the variability and uncertainty in load position have a significant impact on the stress distribution within the optimized layouts. In other words, the resulting optimized layouts under the reliability-based framework exhibit variations according to each value of  $\beta_{target}$ .

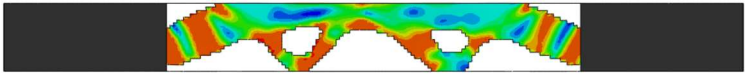
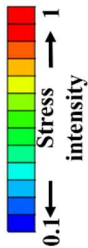
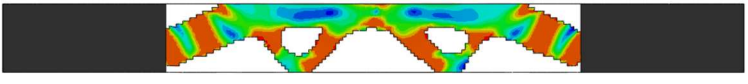
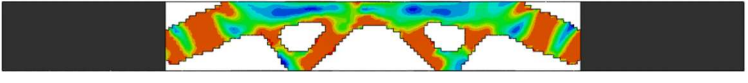
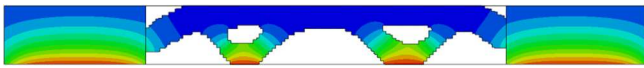
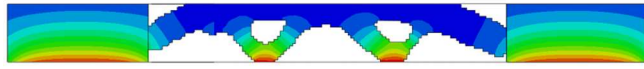
$\beta_{target}$	Resulted layouts	
4.00		
3.50		
3.00		

Table 1: Results of incorporating reliability-based optimization

Exploring the impact of high temperatures involves examining how heat is distributed within the beam. This analysis involves a comparison of the outcomes of the proposed algorithm with those of Habashneh et al.[10]. Table 2 provides a helpful visual representation, illustrating how the temperature is distributed when the lower edge of the flange is exposed to high temperatures. In addition, this presentation highlights the impact of integrating probabilistic design into topology optimization on temperature profiles.

Temperature (°C)	Algorithm	Temperature distribution: Application of fire from the lower flange
800	Proposed method	
	Habashneh et al.[10]	

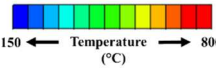


Table 2: Resulted temperature distribution

## 4 Conclusions and Contributions

The study introduces a novel approach for optimizing the topology of steel I-beams under elevated temperatures, integrating reliability-based design principles and thermoelastic-plastic analysis. By employing BESO algorithm within the elastoplastic limit analysis framework, the research demonstrates the effectiveness of this approach in minimizing structural weight while maintaining high performance. Results underscore the significant impact of load position variability on stress distribution within optimized layouts, emphasizing the importance of reliability-based optimization in ensuring structural robustness and safety. Additionally, analysis of temperature distribution contours reveals insights into the thermal behavior of optimized beams, highlighting the practical implications of integrating probabilistic design into topology optimization. Overall, the study contributes to advancing knowledge in structural design for high-temperature environments, offering valuable insights for engineers and researchers in the field.

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