Proceedings of the Fifteenth International Conference on
Computational Structures Technology
Edited by: P. Iványi, J. Kruis and B.H.V. Topping
Civil-Comp Conferences, Volume 9, Paper 4.3
Civil-Comp Press, Edinburgh, United Kingdom, 2024
ISSN: 2753-3239, doi: 10.4203/ccc.9.4.3

Topology Optimization of Strain Energy Constrained 2D Elasto-Plastic Truss

H. Shi[1] and F. J. Montans Leal[1,2]

[1]Escuela Técnica Superior de Ingeniería Aeronáutica y del Espacio, Universidad Politécnica de Madrid, Spain
[2]Department of Mechanical and Aerospace Engineering, University of Florida, Florida, USA

Abstract

Topology optimization in energy-absorbing structures, particularly in the context of elasto-plastic truss elements, presents a crucial avenue for engineering innovation. This paper extends from a one-dimensional toy model to a two-dimensional perfect elasto-plastic truss structure based on evolutionary structural optimization, examining diverse design variables, optimization strategies, structural sizes, and responses to energy absorption thresholds. This study reveals the superiority of certain design variables and optimization strategies, while highlighting challenges such as size dependency and mesh sensitivity. Furthermore, we unveil the nuanced impact of different energy distribution strategies on final topology structures. This work not only enriches our understanding of topology optimization for energy absorption but also inspired for future research and refinement in this domain.

Keywords: topology optimization, energy absorption, elasto-plasticity, truss, strain energy, evolutionary structural optimization

1 Introduction

Energy absorption stands as a pivotal application domain for functionally graded metamaterials, where the paramount objective revolves around crafting structures that are not only lightweight but also proficient in meeting stringent energy absorption requisites, which is the primary objective in engineers' topology optimization designs [1]. Given that a significant portion of energy absorption occurs in plasticity, it becomes imperative to pivot towards employing elastoplastic constitutive models rather than adhering solely to linear elastic models. This strategic shift is pivotal in the design of structures engineered for energy absorption.

Against this backdrop, our paper aims to extend from a one-dimensional (1D) toy model to a two-dimensional (2D) truss structure. We dissect various facets including different design variables, optimization strategies, structural sizes, responses to energy absorption thresholds, and the ramifications of employing diverse energy distribution strategies on the ultimate topology structure. By traversing this multifaceted terrain, we aim to illuminate the nuanced interplay between these variables and strategies, thereby enriching our understanding of the topology optimization landscape in the context of energy-absorbing structures.

Here we demonstrate our idea as a 1D toy example. As shown in Fig.1, n numbers of perfect elastoplastic 1D bar elements are equally spaced with distance d. One node of the bars is fixed to the ground and another node connects to a rigid body. The angle between the rigid body and the horizontal ground is θ. The rigid body was applied a uniform downward force, and the plastic energy absorption of the bar system should absorb a certain amount of work U^*. Neglect the Poisson's effect and energy dissipation.

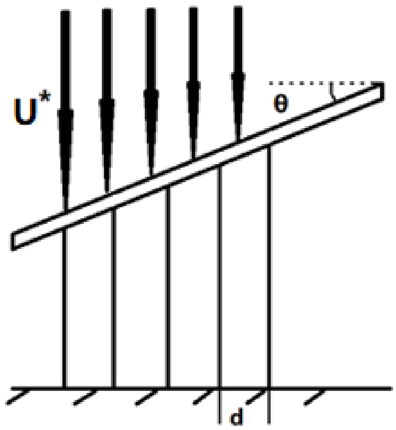

Figure 1. Scheme of 1D toy example.

With the given distance d, inclined angle θ, initial length of the bar L_0, elastic modulus E, strain of yield point ε_y, strain of fracture point ε_u, energy absorption threshold U^*, we want to minimize the total length of bars subjected to the following constrains: no bar reaching fracture point; at least one bar is not yielded; the total (or plastic) strain energy of the bar system is over than the energy absorption threshold. The 1D toy example shows the main idea of the plastic strain energy constrained algorithm. In the following of the paper, we will discuss 2D truss structure.

2 Optimization formulation

2.1 Optimization Problem Statement

In 2D truss structure, the optimization problem can be described as:

$$minimize\colon \sum_{i=1}^{N} W(x_{\mathrm{i}})$$

$$\begin{aligned} subject\ to&: U^* - \sum_{i=1}^{N}(U_{e,i} + U_{p,i}) \leq 0 \\ &x_{\min} \leq x_{\mathrm{i}} \leq 1 \\ &\forall \varepsilon_i < \varepsilon_u \\ &\exists \varepsilon_i < \varepsilon_y \end{aligned}$$

Where $W(x_{\mathrm{i}})$ is the weight function of the design variables x_{i} for ith truss members, two different design variables the cross-section area A and elastic modulus E are discussed. Besides, U^* is the threshold of energy absorption, $U_{e,i}$ and $U_{p,i}$ are the elastic and plastic strain energy of the truss members, N is the total number of the truss members, $x_{\min}$ is a small value representing the void element, ε_u is the strain in fracture point, ε_y is the strain in yield point.

2.2 Geometrical property and constitutive model of truss element

The 2D truss structure is assembled by a certain amount of truss element. Each truss element contains four nodes and each pair of nodes in the element are connected by perfect elastoplastic truss member. To simply the problem we consider the mechanical behaviour of truss members as bars, as well as neglect the Poisson's effect, energy dissipation and geometrical nonlinearity. The geometry and constitutive model of the truss are shown in Fig.2(a) and (b) respectively. $E = 1e8, \varepsilon_y = 0.2\%, \varepsilon_u = 2\%$.

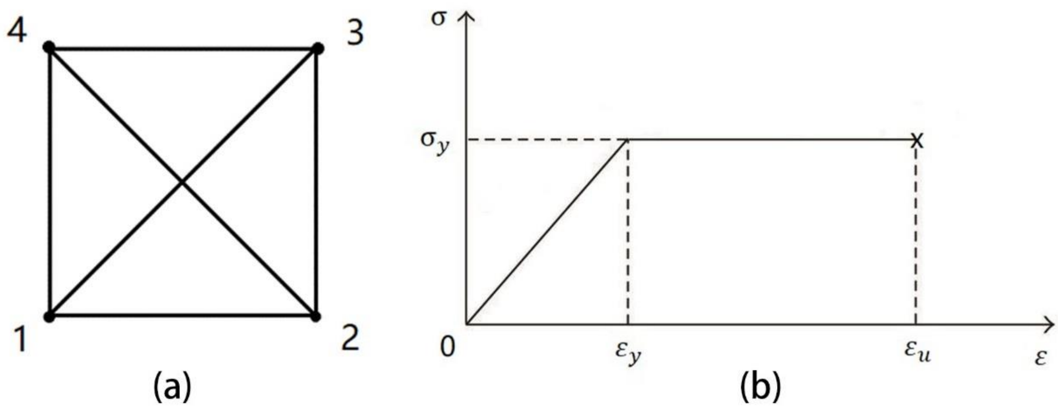

Figure 2: (a) Geometry of a 4-nodes truss element; (b) perfect elastoplastic model of truss member.

2.3 Virtual stiffness method of finite element analysis

To solve the nonlinear problem, the penalty method is used to enforce the increment of Dirichlet boundary conditions transform to approximate Neumann boundary conditions [2]. In this method, a virtual spring with very high stiffness is parallelly connected to the nodes where the Dirichlet boundary conditions are prescribed. When the stiffness of the virtual spring much larger than the stiffness of the truss:

$$K_{spring} \gg K_{truss}$$

the original Dirichlet boundary condition u can be replaced by an approximate Neumann boundary condition:

$$F = (K_{spring} + K_{truss}) \cdot u \approx K_{spring} \cdot u$$

2.4 Load cases and thickness visualization

Here we introduce two types of load cases: cantilever and “Half-MBB”. Fig.3 shows the scheme of the load cases.

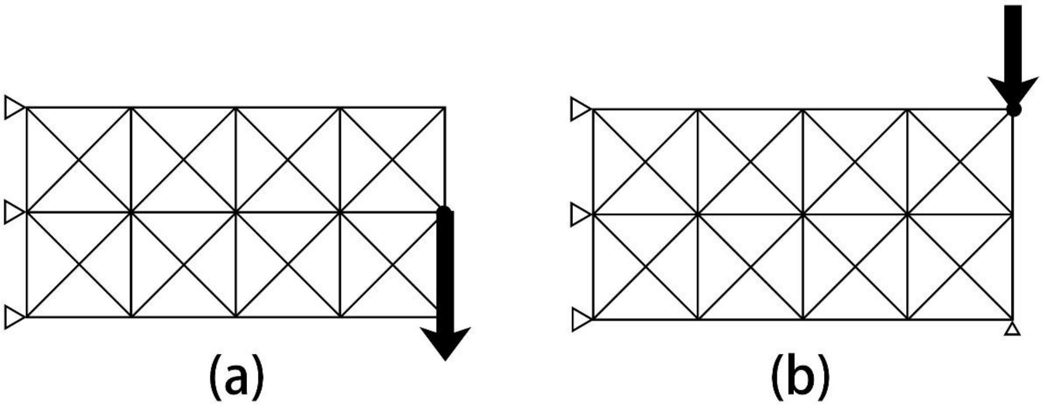

Figure 3: Two load cases. (a). cantilever truss and (b). “Half-MBB” truss.

The thickness of the line represents the percentages remain of design variables. The thicker the line means the truss member is more closing to the initial value, and thus equivalent heavier member. Although the relationship between the elastic modulus and density is not specified here, we can still clearly observe the material distribution from the thickness of the line.

2.5 Optimization algorithm

Evolutionary structural optimization (ESO), based on the philosophy of "survival of the fittest," achieves structural optimization by iteratively removing the elements contributing the least to the global structure [3]. In the following chapter, we discussed two distinct optimization strategies: one involves removing truss members with the lowest strain energy levels, enhancing the overall structural efficiency in absorbing energy; the other entails removing truss members with the highest strain energy levels and its surrounding members will absorb energy, thereby delay the reaching of fracture to enables the algorithm to remove material to the greatest extent possible through as many iterations as possible. To circumvent potential difficulties arising from complete material removal, we employ the soft-kill [4] to eliminate material, wherein a minimum allowable value $x_{\min}$ is set for the optimization variable as a "void" member.

3 Results and discussion

3.1 Design variables

In this part we compared the optimized topology of two different design variables, the cross-section area A vs. elastic modulus E of truss members under cantilever load case, as shown in Fig.4. The results of E as variable have more redundant structure, which are void member in the results of A as variable. This shows under same energy threshold the E as design variable tend to generate the uniformed distribution of material in the design domain, which also proved according to Fig.5. The uniformed distribution also results in a less pronounced final topology. This may contradict our impression of the topology optimization results. Whereas when using A as design variable, the optimization algorithm tends to concentrate the material on key members to form a clear force transmission path. These key members bear most of the strain energy, while the strain energy of other members is relatively small.

Recall that the relation between the elastic modulus and density is not specified, so the final weight of results for E as variable does not mean it will be heavier than the results of A as variable, the thickness only present the percentages remain of design variable. However, since the non-intuitive of topology and the relation of E and weight, variable elastic modulus will make actual manufacturing more complicated, we should carefully choose E as a design variable.

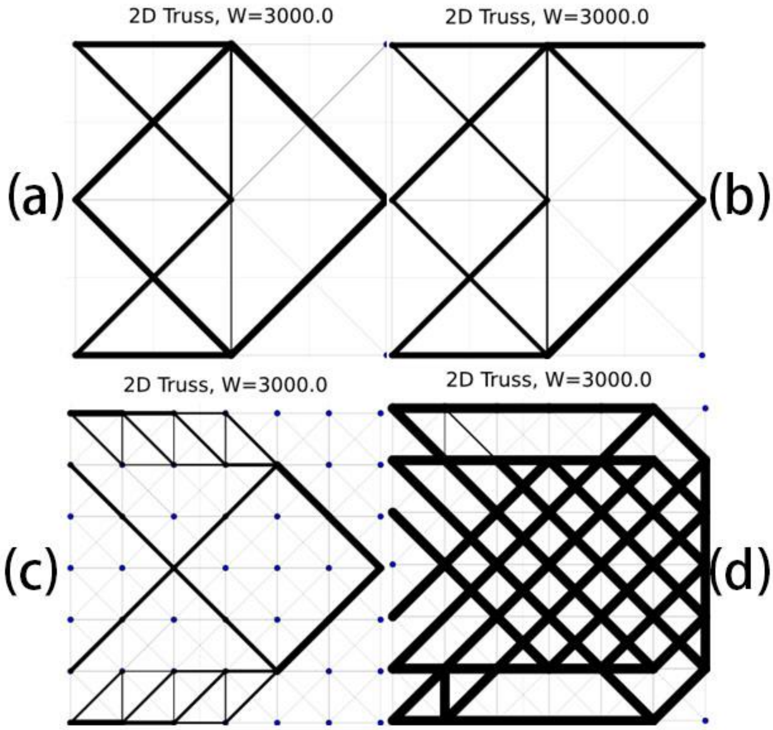

Figure 4: Comparison of design variables of cantilevers: cross-section area A vs. elastic modulus E of truss members. (a). A as variable, 2*2 elements; (b). E as variable, 2*2 elements; (c). A as variable, 6*6 elements; (d). E as variable, 6*6 elements.

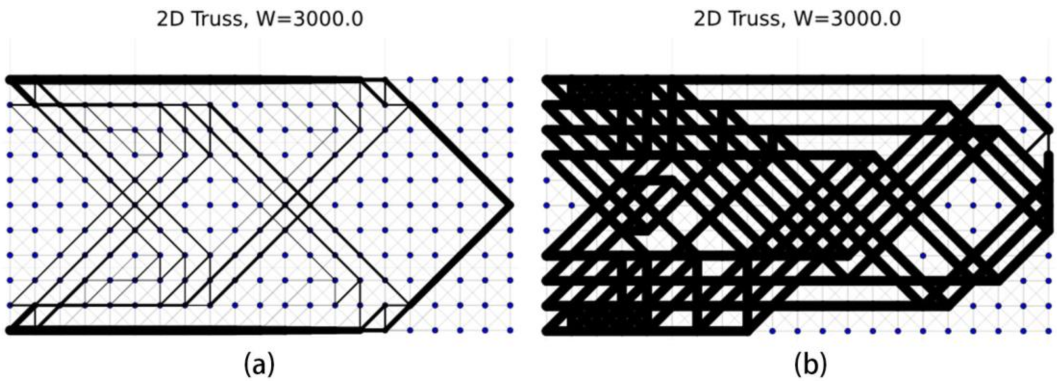

Figure 5: Larger size comparison of design variables of cantilevers: cross-section area A vs. elastic modulus E of truss members. (a). A as variable, 20*10 elements; (b). E as variable, 20*10 elements.

3.2 Optimization strategies

From the above analysis we inspired by the idea of the uniformed distribution of material. The original strategy of ESO algorithm is removing the material of truss members with the lowest strain energy levels (so-called "min" strategy). Since the truss members with concentrated strain energy will finally reaching the fracture point and terminate the optimization algorithm, we reversed the strategy for delaying the reaching of fracture. We remove the truss members with the highest strain energy levels so that its surrounding members will absorb energy (so called "max" strategy). The results of these two strategies are shown in Fig.6. Compared with the min strategy, the results of max strategy are still retained a lot of useless materials in the final topology, and even almost the same as the initial design. Obviously, the optimization results using the max strategy are not satisfactory. We believe that when an efficient member is removed, the inefficient members around it cannot generate new efficient force transmission paths, finally leads insufficient optimization. We only use min strategy in the following research.

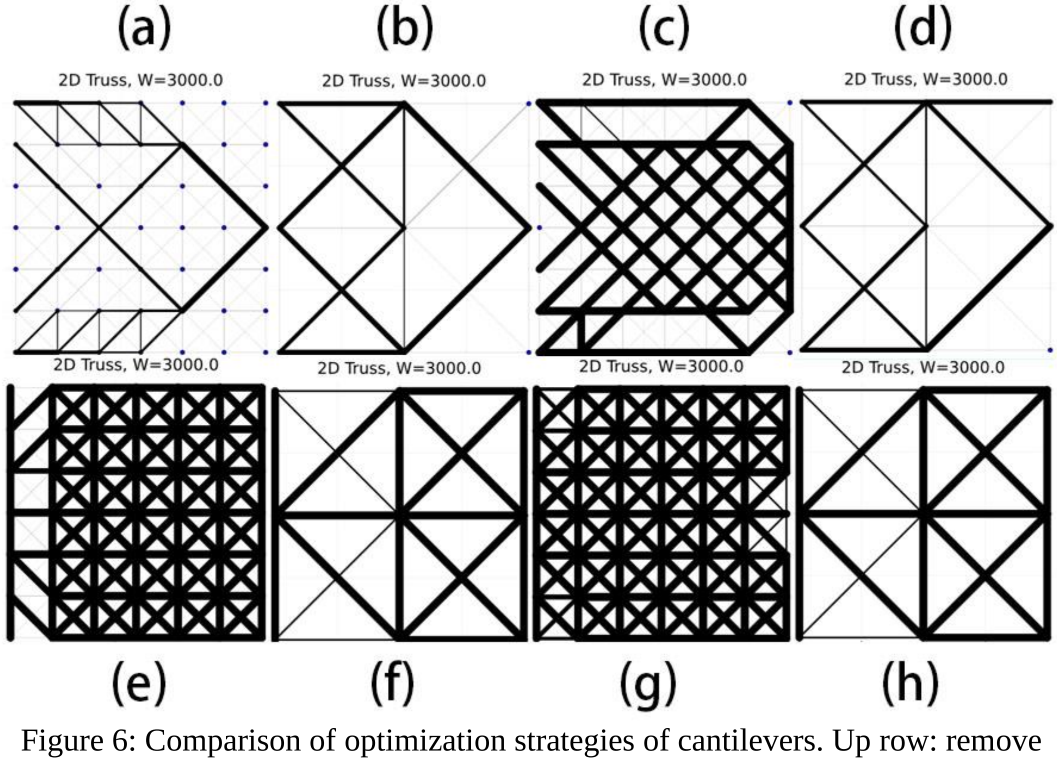

Figure 6: Comparison of optimization strategies of cantilevers. Up row: remove lowest strain energy member (min strategy) vs. down row: remove highest strain energy member (max strategy). (a). A as variable, 6*6 elements, min strategy; (b). A as variable, 2*2 elements, min strategy; (c). E as variable, 6*6 elements, min strategy; (d). E as variable, 2*2 elements, min strategy; (e). A as variable, 6*6 elements, max strategy; (f). A as variable, 2*2 elements, max strategy; (g). E as variable, 6*6 elements, max strategy; (h). E as variable, 2*2 elements, max strategy.

3.3 Energy thresholds

In this section, we demonstrate the topological configuration as the energy threshold increases in Fig.7. In the results of A as variable, as the energy threshold increases, the amount of material remaining increases, and new force transmission paths appear to absorb energy. The case is different in the results of E as variable. Due to the redundant material remains, the thicknesses of the lines have no significant changes, and the topology in general also have minus changes. It furtherly proved the E as variable will cause inefficient optimization as discussed in chapter 3.1.

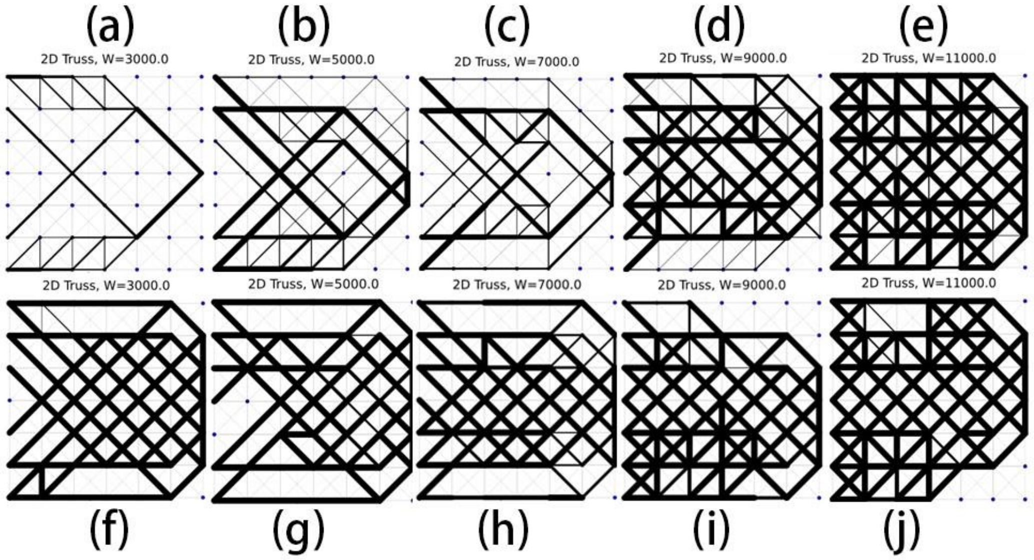

Figure 7: Trends of increasing the energy thresholds of 6*6 elements cantilevers. (a)-(e): A as variable, energy thresholds = 3000, 5000, 7000, 9000, 11000 respectively; (f)-(j): E as variable, energy thresholds = 3000, 5000, 7000, 9000, 11000 respectively.

3.4 Size effect

In this section we want to understand the effect of the structure size (numbers of the truss elements) on the energy absorption. U_p ratio shows how much plastic strain energy occupied in total strain energy. According to our constitutive model, the upper limit of the plastic strain energy that the same amount of material can absorb is 18 times of its elastic strain energy. Hence, U_p ratio can help to evaluate the absorbing efficiency of a structure. Higher U_p ratio means more plastic strain, under same energy threshold the structure requires less amount of material.

We firstly evaluate the size dependency, note that we kept the length as a constant of each element, more element equal to larger size of the global structure. Fig.8 listed 4 different optimization results and its energy contributions. From Fig.8, when we expand the structure size from 2*2 to 6*6 elements, the energy contributions transformed from plastic strain dominated to elastic dominated. The larger cantilever is stiffer than the small one, the stress will be more dispersed, so that the global strain is also much smaller than the small cantilever. Besides, the U_p ratios of A as variable are over than E, again proves the A is more suitable for the design variable.

We kept the design domain as constant to explore the mesh independency. The mesh independency is a necessary check in the algorithm of referenced volume element (RVE) of solid topology optimization, aiming to obtain the optimization results which independent of the mesh density in a certain size of design domain. Although truss elements are different from RVE, the increment of mesh density will introduce more materials, here we still verified the mesh independency to observe the effects, shown in Fig.9. The results showed that the increment of mesh density will greatly change the final topology, caused poor energy absorbing structure with low U_p ratios.

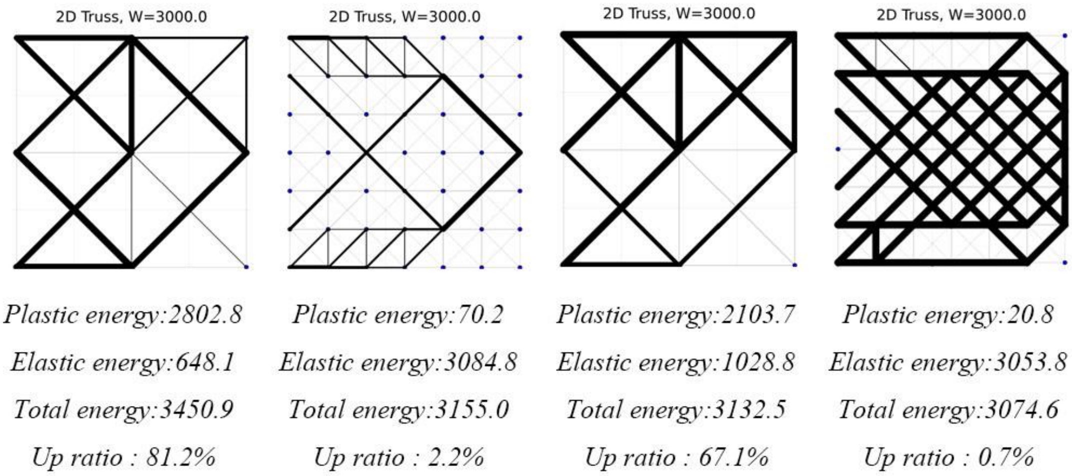

Figure 8: Structure size dependence of topology result under cantilever load case. (a). A as variable, 2*2 elements; (b). A as variable, 6*6 elements; (c). E as variable, 2*2 elements; (d). E as variable, 6*6 elements.

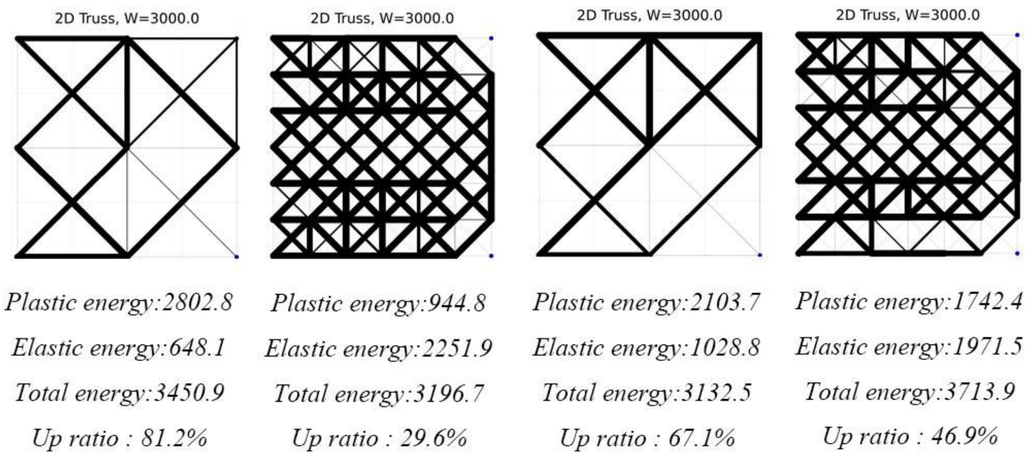

Figure 9: Mesh dependence of topology result under cantilever load case. (a). A as variable, 2*2 elements; (b). A as variable, 6*6 elements; (c). E as variable, 2*2 elements; (d). E as variable, 6*6 elements.

3.5 Energy distribution strategies

In this section we compared the difference between two energy distribution strategies: total strain energy as constrain and only plastic strain energy as constrain. The idea came from the huge ratio of the maximum plastic strain energy to the elastic strain energy that a material can absorb before fracture, as well as the poor U_p ratios in larger size design. To set a fair threshold, we firstly run the total strain energy algorithm and recorded the value of absorbed plastic strain energy, the value will be the threshold of only plastic strain energy algorithm. The results in Fig.10 showed the significant difference of optimal topology. Instead of achieving a similar topological configuration, the algorithm of only plastic strain energy as constrain retained many inefficient truss members.

The algorithm will apply load until the total plastic strain energy reaching threshold from the first iteration. At that time, the low efficient material has not yet been removed, but it might have significant strain especially the truss members closing to the boundary condition (stress concentration). Since the ESO algorithm will remove the truss member which has minimum level of strain energy, the plastic flow will "freeze" in the truss members which are yielded during the first iteration and lead a typical local optimum. The mathematical programming algorithm like method of moving asymptotes might avoid the trap.

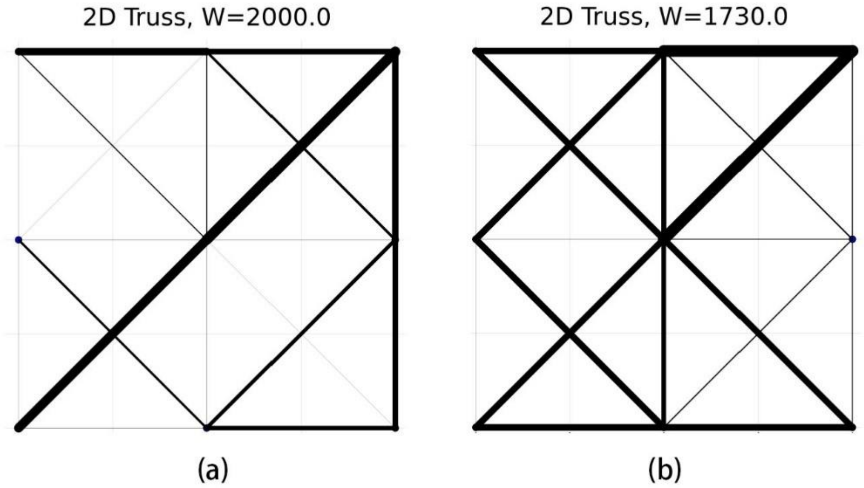

Figure 10: Comparison of different energy distribution strategies of "half-MBB" load case truss structure. (a). total strain energy as constrain; (b). plastic strain energy as constrain.

4 Conclusions and Contributions

In this paper, we delved into a comprehensive exploration of various facets within the realm of topology optimization for strain energy constrained 2D elasto-plastic energy absorbing trusses based on evolutionary structural optimization. Specifically, we scrutinized diverse design variables, optimization strategies, structural sizes, responses to energy absorption thresholds, and the influence of different energy distribution strategies on the final topology structure.

Our findings underscore several key insights. Firstly, we observed that the cross-sectional area of truss members emerges as a more advantageous design variable compared to elastic modulus. Additionally, we determined that the optimization strategy of evolutionary structural optimization (ESO), focusing on removing materials with the lowest strain energy levels, surpasses the efficacy of delaying fracture strategies. However, it is noteworthy that our analysis revealed inherent dependencies on size and mesh within the algorithm, suggesting the necessity for further refinement and enhancement. Furthermore, we discovered that imposing a constraint on total strain energy yields superior outcomes compared to directly constraining plastic strain energy, as the latter tends to retain plastic flow in less efficient materials from the initial iterations. This underscores the potential for exploration with alternative topology optimization algorithms.

Our study elucidates inherent challenges within the topology optimization of energy-absorbing trusses, particularly regarding mesh dependency and the constraints associated with plastic strain energy. By shedding light on these issues, our work not only paves the way for future research but also offers a discerning comparison of the effects of design variables and optimization strategies.

Acknowledgements

This project has received funding from the European Union's Horizon 2020 research and innovation programme under the Marie Skłodowska-Curie Grant Agreement No. 956401

References

[1] M. Mohsenizadeh, F. Gasbarri, M. Munther, A. Beheshti, K. Davami, "Additively-manufactured lightweight Metamaterials for energy absorption.", Materials & Design, 139, 521-530, 2018.

[2] J. W. Barrett, C. M. Elliott, "Finite element approximation of the Dirichlet problem using the boundary penalty method.", Numerische Mathematik, 49, 343-366, 1986.

[3] Y. M. Xie, G. P. Steven, "Evolutionary structural optimization for dynamic problems.", Computers & Structures 58, no. 6, 1067-1073, 1996.

[4] X. Huang, Y. Xie, "A further review of ESO type methods for topology optimization.", Structural and Multidisciplinary Optimization 41, 671-683, 2010.