Proceedings of the Fifteenth International Conference on
Computational Structures Technology
Edited by: P. Iványi, J. Kruis and B.H.V. Topping
Civil-Comp Conferences, Volume 9, Paper 4.2
Civil-Comp Press, Edinburgh, United Kingdom, 2024
ISSN: 2753-3239, doi: 10.4203/ccc.9.4.2

A Combinatorial Analysis for the Assessment of the Optimal Tie Rods' Configuration in Historical Masonry Buildings

**M. Buzzetti[1], R. Cucuzza[2], M. Domaneschi[2],
M. Movahedi Rad[3], G. C. Marano[2] and G. Milani[1]**

**[1] Department of Architecture, Built Environment and
Construction Engineering, Politecnico di Milano, Italia
[2] Department of Structural, Geotechnical and Building
Engineering (DISEG), Politecnico di Torino, Torino, Italy
[3] Department of Structural and Geotechnical Engineering,
Széchenyi István University, Győr, Hungary**

Abstract

Retrofitting and strengthening measures are crucial for preserving historical masonry structures against earthquakes. Tie rods are often the preferred solution due to their effectiveness, feasibility, and cost efficiency. Additionally, it allows to minimise architectural impact, especially for historical buildings. A comprehensive combinatorial analysis has been conducted within a limit analysis-based approach. Such an approach discretises the structure using hexahedron elements with quadrilateral interfaces, subjecting it to various input loads including seismic ones. The procedure explores all possible retrofitting configurations by evaluating different arrangements of the tie rods by varying the anchoring points of the bars connecting opposite facades of the building. A tree-shaped multi-stage algorithm is adopted to reduce the research domain and identify the optimal retrofitting strategy. Effectiveness of each configuration is evaluated using a mechanical parameter — the velocity indicator — linked to power dissipation on cracks. The methodology is tested on the ex-monastery of Santa Maria della Pace in Piacenza (Italy), featuring a single-nave church with an embedded bell tower.

Keywords: Heritage, Historical masonry, Limit analysis, Tie rods, Parametric analysis, Seismic assessment

1 Introduction

Masonry structures, especially those of historical significance, pose unique challenges in seismic assessment and retrofitting because of their complex geometries, material properties and intrinsic uncertainty due to the limited knowledge and lack of historical data. Understanding the behaviour of these structures under seismic loading is crucial for preserving the cultural heritage and ensuring public safety [1, 2].

This paper presents a first step for the assessment of the optimal distribution of tie roads for the seismic retrofit of masonry structures, particularly focusing on the historical contexts. Central to this study is the use of a preliminary combinatorial analysis to assess the effectiveness of different tie rods positioning, to evaluate the structural response under increasing horizontal loads, which is particularly relevant for ancient masonry structures characterised by arches and vaults [3–5].

The investigation employs a 3D discrete element model of the church of the ex-monastery of Santa Maria della Pace (Figure 1) in Piacenza (Italy), with the aim of developing the limit analysis of the nonlinear model of the building subjected to increasing monotonic horizontal loads. The analysis considers the different combinations of tie rods' connection conditions aiming to assess the global response of the church as part of the building aggregate, and the local response of its different portions such as the facade and the bell tower. In this way, the in-plane behaviour and potential failure mechanisms for each retrofitting solution are investigated and the effect of different tie rod configurations is evaluated. Emphasising the importance of accurately modelling both geometry and material behaviour [6], this study aims to assess the effectiveness of the tie-rods retrofitting solution with respect to the seismic vulnerability mitigation, and determine the most effective retrofitting intervention through a combinatorial approach [7]. Special attention is paid to the connections between the walls and the tie rods, as these factors significantly influence the safety of masonry structures during seismic events [8, 9].

The study delves into the use of an upper bound limit analysis-based approach to capture the structural behaviour of the church. The structure is discretized in rigid blocks where the inelastic deformation can occur only at the elements' interfaces. The approach allows the evaluation of the collapse acceleration by evaluating the critical collapse mechanisms and the necessity for retrofitting interventions. Through the proposed methodology, this paper offers valuable insights into the seismic vulnerability

Figure 1: The ex-monastery of Santa Maria della Pace in Piacenza (Italy): (a) global view, (b) 3D view of the church including the bell tower

assessment and mitigation of masonry structures in historical contexts through tie-rods installation.

The paper is organized as follows: Section 2 introduces the methodology of the limit analysis and the implementation criteria of the combinatorial analysis. Section 3 is devoted to the definition of the case study while section 4 offers a detailed description of the F.E. model and a comprehensive discussion of the results obtained from the analyses. Finally, conclusions and future developments are pointed out in Section 6.

2 Methodology

In the context of restoring an ancient historical building, such as the ex-monastery of Santa Maria della Pace in Piacenza, the combinatorial logic plays a pivotal role in the design of tie-rods used to reinforce the structure [9]. They are arranged according to specific constraints, such as their limited application to the upper part of the building, where their performance are maximized, and their anchoring to parallel walls at the same height. By adopting a combinatorial approach, all feasible tie-rod configurations, which respect geometric and structural constraints, are evaluated aiming to identify the optimal retrofitting strategy for ensuring maximum structural stability. The best combination, assumed to be composed by a maximum of two tie-rods, is defined by the highest mean value of the collapse load in the four horizontal loading directions (X and Y positive and negative directions) under the most severe distribution of horizontal forces (G1 triangular or G2 uniform, as defined in the Italian building code [10]). In this sense, the combinatorial problem has been practically solved by means of a parametric analysis where the extreme edge of the rods dynamically change during the process.

The methodology is therefore based on a parametric analysis in which the effect resulting from the position of the tie rods, for each considered configuration, has been investigated based on the corresponding collapse load. At this stage, the cross-section of the chains and other geometric and/or structural constraints have not been taken into account. Only the optimal layout of the tie rods has been investigated in this preliminary work.

The adopted methodology provides the discretization of the structure in infinitely resistant hexahedron elements. Materials constituting historical masonry structures are assumed rigid-perfectly plastic characterized by infinite ductility while inelastic deformations can occur only at elements' interfaces. Therefore, the structure is discretized into distinct elements and the problem belongs to classic limit analysis. Among the different outputs, the collapse load and the active failure mechanism can be evaluated. Under such hypotheses, the solutions found with the upper bound and lower bound theorems coincide since the plasticity is concentrated in a finite number of interfaces. Since the limit analysis problem formulation can be done both from a kinematic and static point of view, the kinematic approach is adopted due to its simplicity. Then adopting the formulation of the self-dual linear programming problem, the static counterpart can be derived. The primal variables of the problem consist of six unknowns per hexahedron, in particular centroid velocities along the reference axes and the rotation rates around the centroid. External forces are assumed to be only volume forces applied at the centroids of the elements, consisting of a set which is independent from the collapse multiplier and another set dependent on it.

Considering that plastic dissipation is allowed only at elements' interfaces, the jump of velocities on the generic interface between two elements should be evaluated in order to preserve plastic compatibility. For each interface it is assumed a constant stress state, meaning that the compatibility constraint on the jumps of velocities is imposed on a single point of the interface.

The interface material was assumed to obey a Mohr-Coulomb failure criterion with tension and compression cutoff, characterised by a tensile strength (f_t), a compressive strength (f_c), a cohesion (c) and a friction angle (ϕ).

At elements' interfaces, a set of linear inequalities can be written to define the plastically admissible strength domain. Considering an associate flow rule, for each element interface the jump of velocities is linked to the plastic strain rate vector, leading to a set of linear equations. But since the plastic strain rate vector is non-negative, there is a set of further inequalities for each interface. In any upper bound limit analysis, the normalization condition is needed to identify one failure mechanism among the infinite set of homothetic deformed shapes. This constraint - generally imposed assuming that the power dissipated by the external loads, expressed in function of the collapse multiplier, is unitary when the collapse multiplier is equal to one - leads to another set of inequality constraints. Given the normalisation condition, the collapse multiplier is simply computed as the difference between the internal power and the power expended by the external loads independent of the load multiplier. In the framework of the upper bound theorem, this difference is the objective function to minimise. Finally, standard inequalities involving generalised velocities should be written to impose the boundary conditions of those elements whose nodes are externally constrained. Tie rods are modelled using rigid-plastic truss elements anchored at hexahedron centroids. In the proposed methodology, the presence of tie rods leads to a set of linear equations constraining the velocities of the elements connected by the tie rods. In order to find the collapse load multiplier and the active failure mechanism, a linear programming problem should be solved, considering the relationship between jump of velocities and plastic multipliers on all interfaces, the normalization condition of the failure mechanism, boundary conditions, presence of tie rods and the objective function to minimise. This method is implemented with a script in the software *MATLAB*. For more information about the analytical procedure, the reader is referred to [11].

In Figure 2 a simple representation of the adopted case study is shown for clarity purposes. The schematic of the church reports for simplicity the four walls and an indication of the roof. The methodology works by defining two sets of possible ends of the tie-rods, that should be on parallel walls at the same height. For this reason, the elements of the upper rows of the facing walls are highlighted, naming A_h and B_k the *h-th* and *k-th* elements belonging to the two walls. The procedure selects randomly four of these elements for all the possible configurations defining two tie-rods connecting the opposite facades. As depicted in Figure 2, the two edges of the tie rods labelled as A_2 and B_9 for the first chain and A_6 and B_2, for the second one, respectively, are selected for this specific combination.

For each combination, the collapse load, calculated as the ratio between the base shear and the vertical load, is evaluated by means of the upper bound limit analysis approach according to the four loading directions coherently to the selected horizontal forces distribution. Because of the large dimension of the research domain, the computation demand has been reduced by adopting a tree-shaped multi-stage algorithm. First, the structure is discretized using big elements (rough mesh) and the best 2 or 3 combinations are identified. Then the mesh is refined and the set of elements potentially selected for the installation of the chain, is limited to the area identified in the previous step. The most effective combination is the one defined by the highest mean value of the collapse load among all the four investigated loading directions.

This is just the initial step in a broader process. To further optimise the project and solve complex structural engineering problems, it is necessary to resort to the use of artificial intelligence algorithms or intelligent routines. The transition towards metaheuristic algorithms is motivated by several factors. Firstly, they allow for better optimisation of load and computational times, reducing manual work and speeding up the design process. Additionally, they enable obtaining more cost-efficient solutions by considering available materials and resources, as well as maximising structural

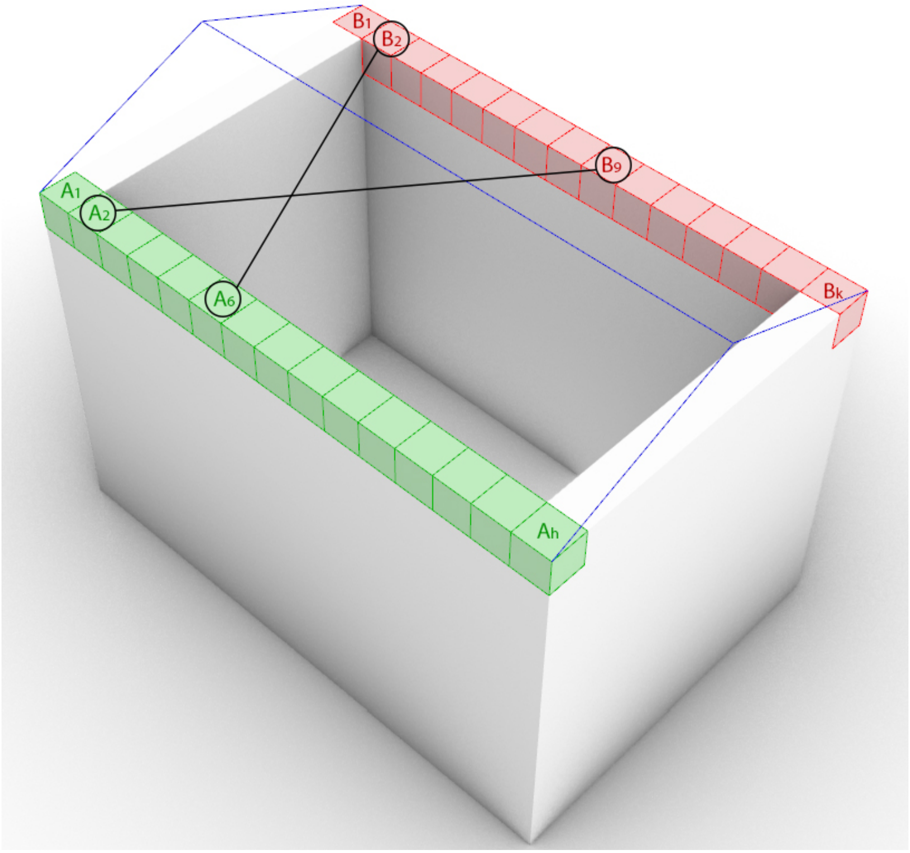


Figure 2: The combinatorial approach for the tie rods optimisation procedure

performance to ensure the building's long-term safety.

The integration of combinatorial analysis with advanced artificial intelligence algorithms represents a significant advancement in the field of structural engineering, offering a novel optimization framework where the optimal design of the retrofitting scenario can be obtained automatically independently of the specific target study. This interdisciplinary approach not only enhances the efficiency and effectiveness of restoration projects but also contributes to the broader understanding and application of mathematical and computational techniques in structural conservation.

3 The ancient masonry structure in aggregate

The ex-monastery of Santa Maria della Pace in Piacenza, Italy, was built by the Benedictine nuns in the 16th century and has undergone several changes over the subsequent centuries. The surviving portion of the original structure has not been significantly altered. The monastery is characterised by a cloister layout with two levels, featuring cross vaults covering the cloister and corridors on the first floor, and rooms mainly covered by cloister vaults, with some rooms having timber beam and joist decks. The structure also includes a pitched roof with varying heights, depending on the elevation geometry of different parts of the building. The bearing structure is made of timber beams and joists and is covered by tiles. Additionally, the structure is considered a building aggregate due to its confinement by adjacent buildings on two sides, as well as the presence of a church and a bell tower within the complex. The church has a large single nave covered by a barrel vault with reinforcing arches as depicted in Figure 3, while the bell tower is a slender hollow structure [8]. The church of the ex-monastery was chosen as a case study for the development of the methodology due to its simple geometry. Since the preliminary stage of the research, the church connected with the bell tower is studied as an isolated building neglecting any interaction effect between the two structures. In future developments, also the constraints provided by adjacent buildings will be considered.

The structure is made of masonry constituted by regular solid clay bricks, and the mechanical properties have been defined based on the level of knowledge reached only by visual inspections, namely LV1 by the Italian building code [10] . The masonry material properties have been evaluated using the Masonry Quality Index (MQI) method, which leads to the assessment of the mechanical parameters of masonry, as shown in [8]. The parameters required for the definition of the Mohr Coulomb strenght criterion used in the proposed methodology are tensile strength $f_t = 0.07\ MPa$, compressive strength $f_c = 2.5\ MPa$, cohesion $c = 0.07\ MPa$ and friction angle $\phi = 30°$.

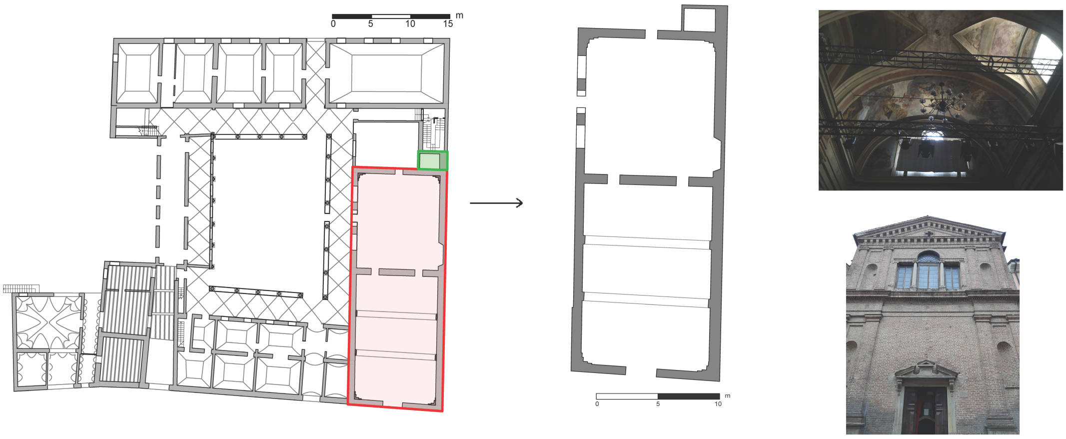


Figure 3: The church of the ex-monastery of Santa Maria della Pace

4 The structural model

The structural model of the church is created starting from a 3D geometric model built in *Revit* and subsequently imported in the commercial software *Straus7*, that is used only to create the mesh defined by hexaedron (Hexa8) elements. Since after a visual inspection it was assumed that the barrel vault has only an aesthetic function and not a structural one, it is disregarded in the structural model. The upper bound limit analysis is carried out in the software *MATLAB* starting from the mesh defined in *Straus7*. The material properties, specified in the previous section, are directly given as input in the *MATLAB* script.

In order to follow the methodology proposed, which involves a tree-shaped multi-stage algorithm, two structural models are built. The first is characterised by a rough mesh (Figure 4a), instead the second one is characterised by a finer mesh (Figure 4b).

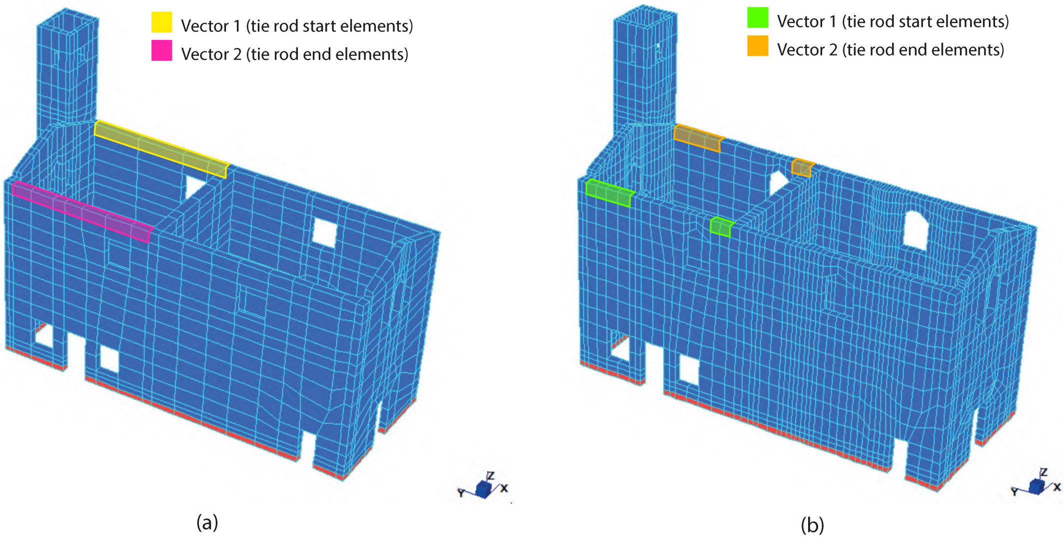


Figure 4: Structural model by adopting a (a) rough mesh (1^{st} step) and (b) finer mesh (2^{nd} step). Masonry elements potentially selected as extreme of the tie rods are highlighted for each analysis step.

5 Results

First of all the seismic behaviour of the church is evaluated without the presence of tie rods. Results are shown in Figure 5 for both the load cases (G_1 and G_2) applied in both positive and negative X and Y directions. The worst load configuration is G_1 (triangular distribution of horizontal forces), which leads to the lowest values of collapse loads in all directions. Moreover, the most vulnerable parts of the church are the bell tower and the room adjacent to it. For these reasons, the following analyses are performed according to the most severe distribution G_1 and, as a first intervention, a combination of two tie rods is searched for the room adjacent to the bell tower.

According to the adopted tree-shaped multi-stage algorithm, two vectors of elements for the rough mesh are identified as possible starting and ending points of the tie rods, as shown in Figure 4a. Having 4 elements in Vector 1 and 7 elements in Vector 2, the total number of possible combinations is 252. In *MATLAB* software, limit analyses are performed for each pair of tie rods in the four loading directions. The best three combinations, according to the highest mean values of collapse load in the four directions, have been identified and they are depicted in Figure 6. Starting from these combinations, in the finer mesh, the area of investigation is restricted to a limited region. The corresponding elements in the model, characterised by a finer mesh, have been selected (see Figure 4b) allowing the reduction of the computational effort. During the elements selection, Configuration 2 has been disregarded since one of the two tie rods is anchored above the window. Indeed, instead of evaluating 15840 combinations, only 300 combinations are analysed since 5 and 6 elements have been identified for Vector 1 and Vector 2, respectively. Once again, the analyses are performed for each pair of tie rods. The best solution is shown in Figure 7, as well as the active mechanism and collapse loads (ag/g), expressed in terms of the ratio between base shear and vertical load in all directions.

The best layout identified for a pair of tie rods placed in the smaller room is characherised by two crossing tie rods. Comparing the collapse load found with and without tie rods, it is possible to see that a significant increase of the collapse load is reached only for load distributions in X direction. The bell tower is still vulnerable against horizontal loads. Besides the tie rods configuration found, further tie rods layouts should be studied for strengthening the bell tower.

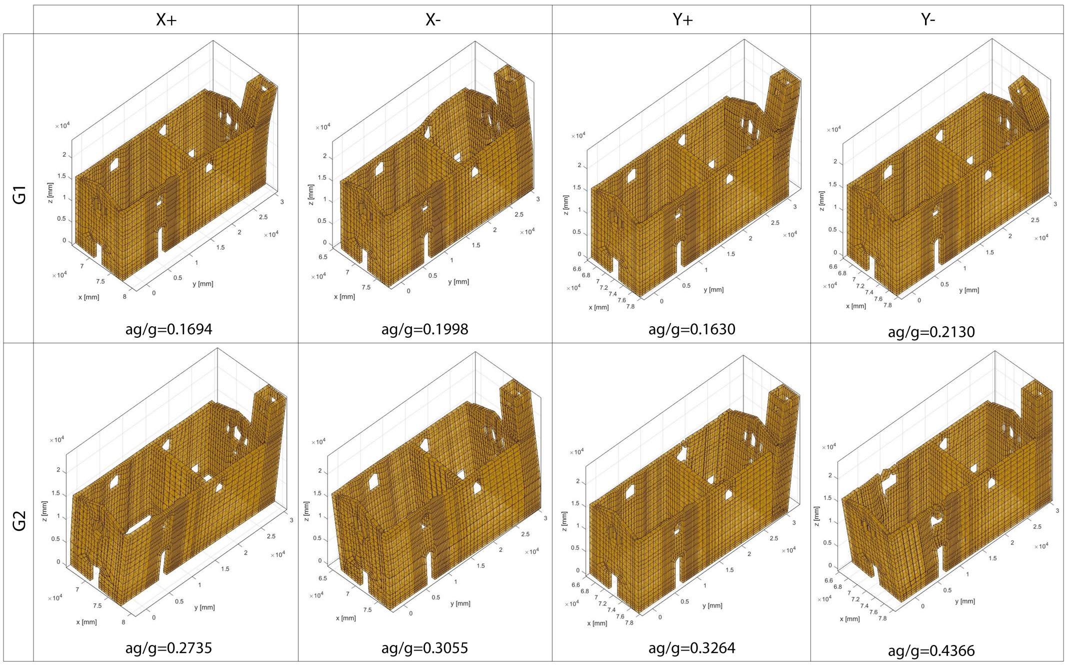


Figure 5: Limit Analysis results without tie rods for loading distribution G_1 and G_2

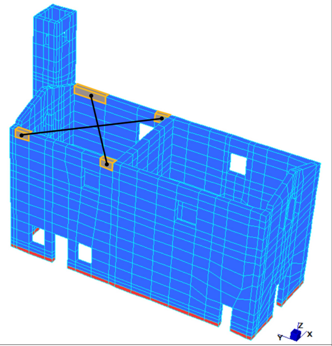
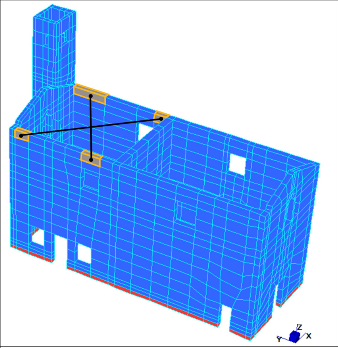
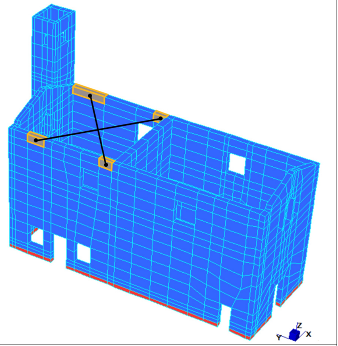
	Combination 1	**Combination 2**	**Combination 3**
ag/g			
X+	0.1881	0.1879	0.1881
X-	0.2435	0.2430	0.2416
Y+	0.1682	0.1682	0.1682
Y-	0.2163	0.2163	0.2163
mean	**0.2040**	**0.2038**	**0.2035**

Figure 6: Best three tie rods configurations found with the rough mesh

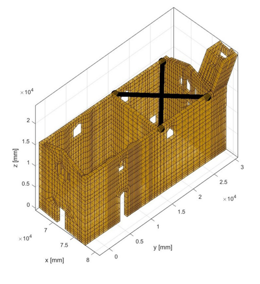
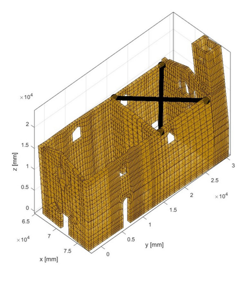
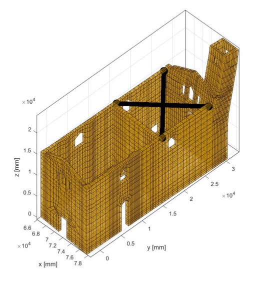
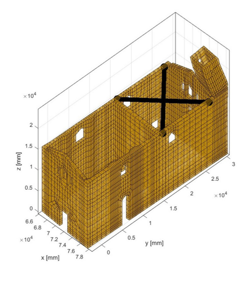
	X+	X-	Y+	Y-
G1	 ag/g=0.1781	 ag/g=0.2205	 ag/g=0.1631	 ag/g=0.2147

Figure 7: Most effective layout for a pair of tie rods

6 Conclusions

An upper bound limit analysis approach with combinatorial optimization is used to find the best layout of a pair of tie rods for strengthening historical masonry structures. An algorithm identifies all the possible layouts starting from two vectors, entered manually, containing possible starting and ending points of tie rods. The approach is based on a tree-shaped multi-stage algorithm to reduce the computational effort. Indeed, at the beginning, the best 2 or 3 configurations are identified using a rough mesh, which involves a limited number of possible tie rod layouts. Subsequently, a finer mesh is used to identify the final tie rod configuration, but limiting the area of possible tie-rods anchoring positions based on the previous results.

The procedure has been tested on the church of the ex-monastery of Santa Maria della Pace in Piacenza (Italy). Since the study is at an initial stage, even though the church belongs to a building aggregate, it has been considered as an isolated building. Further investigations considering external constraints provided by adjacent buildings will be carried out in future studies.

A first configuration of tie rods has been identified for the most vulnerable portion obtained from the analysis. The layout is characterised by crossing tie rods, which significantly increases the collapse load of the structure when loaded in both positive and negative X directions. The bell tower remains the most vulnerable part, therefore, besides the layout identified, further tie rod configurations should be installed to guarantee the stability of the bell tower.

This procedure outlines the initial step of a process aimed at optimizing the design of strengthening interventions with tie rods. However, it emphasises the importance of incorporating artificial intelligence algorithms, particularly genetic algorithms, to achieve optimal solutions, reduce manual work and computational effort, and speed up the design process. These algorithms also facilitate cost-efficient solutions by considering available materials and maximising structural performance for long-term safety.

In future developments of this work, aiming to reduce the computational effort of the current approach and improve the exploration efficiency of the automatic routine, broader investigations will include the use of metaheuristic algorithms like the Genetic Algorithm by considering other structural, economic and environmental indicators like structural cost (i.e. weight), economic costs, and Carbon footprint as target functions of the optimization problem statement [12]. In addition, a more accurate modelling of the confinement provided by neighbouring portions as well as the preliminary design of the tie rods will be investigated in future developments of the current research.

Acknowledgements

This study was partially developed within the research activities carried out in the framework of the 2022-2024 ReLUIS Project—WP5.3 (Coordinators—Andrea Prota, Francesca Da Porto). This project was funded by the Italian Department of Civil Protection. Additionally, the research leading to these results has received funding from the European Research Council under the Grant agreement ID: 101007595 of the project ADDOPTML, MSCA RISE 2020 Marie Skodowska Curie Research and Innovation Staff Exchange (RISE).

References

[1] T. H. Tuan, S. Navrud, Capturing the benefits of preserving cultural heritage, Journal of cultural heritage 9 (3) (2008) 326–337.

[2] M. Acito, M. Buzzetti, C. Chesi, E. Magrinelli, G. Milani, Failures and damages of historical masonry structures induced by 2012 northern and 2016–17 central italy seismic sequences: Critical issues and new perspectives towards seismic prevention, Engineering Failure Analysis 149 (2023) 107257.

[3] A. Formisano, N. Ademovic, An overview on seismic analysis of masonry building aggregates, Frontiers in Built Environment 8 (2022) 966281.

[4] A. Penna, A. Rosti, M. Rota, Seismic response of masonry building aggregates in historic centres: observations, analyses and tests, in: Seismic behaviour and design of irregular and complex civil structures IV, Springer, 2022, pp. 19–36.

[5] F. Clementi, V. Gazzani, M. Poiani, S. Lenci, Assessment of seismic behaviour of heritage masonry buildings using numerical modelling, Journal of Building Engineering 8 (2016) 29–47.

[6] R. Cucuzza, A. Aloisio, R. Di Bari, M. Domaneschi, Vulnerability assessment and lifecycle analysis of an existing masonry arch bridge, Engineering Structures 302 (2024) 117422.

[7] R. Cucuzza, G. C. Marano, Structural optimization through cutting stock problem, in: Italian Workshop on Shell and Spatial Structures, Springer, 2023, pp. 210–220.

[8] M. Acito, M. Buzzetti, G. A. Cundari, G. Milani, General methodological approach for the seismic assessment of masonry aggregates, in: Structures, Vol. 57, Elsevier, 2023, p. 105177.

[9] M. Rettl, M. Pletz, C. Schuecker, Evaluation of combinatorial algorithms for optimizing highly nonlinear structural problems, Materials & Design 230 (2023) 111958.

[10] DM17/01/2018, Aggiornamento delle "Norme tecniche per le costruzioni", Italian Ministry of Infrastructures and Transportation, Rome, Italy, 2018, in Italian.

[11] P. Wang, G. Milani, Specialized 3d distinct element limit analysis approach for a fast seismic vulnerability evaluation of massive masonry structures: Application on traditional pagodas, Engineering Structures 282 (2023) 115792.

[12] R. Cucuzza, A. Aloisio, M. Movahedi, M. Domaneschi, Constructability-based optimization for steel structures: From truss beams to large-scale industrial buildings (approaching final editor decision), Automation in Construction (2024).