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UPM Based Topology Optimisation of Nonlinear Materials

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Abstract

In this paper, we present topology optimisation formulation considering material and geometrical nonlinearities based on the Updated Properties Model (UPM). We use the Saint Venant and neo-Hookean strain energy for physical nonlinearity and the Green-Lagrange strain tensor for geometrical nonlinearity. The suggested method can be applied to the effective design of hyperelastic metamaterials, with the primary potential applications being in biomedical and soft robotic systems.

Keywords: topology optimisation, updated properties model, hyperelastic materials, soft metamaterials

1 Introduction

Recent years have seen a significant increase in interest in the application of structural optimisation to mechanical parts [1,2]. There are three types of structural optimisation: size optimisation, shape optimisation and topology optimisation. Among them, topology optimisation is arguably the one that is utilised in academic and industrial settings the most. To now, various topology optimisation approaches have been introduced and programmed, for example, the Solid Isotropic Material with Penalisation (SIMP) method [3], the Level Set-based method [4], the Evolutionary Structural Optimisation (ESO) method [5] and the Bi-evolutionary Structural Optimisation (BESO) method [6].

More recently, a new scheme for the optimisation of the topology has been introduced by Saucedo-Mora et al [7,8], who have named it the Updated Properties Model (UPM). Unlike the most common topology optimisation techniques like SIMP, the UPM directly tackles the mechanical properties instead of relying on heuristic filters to prevent numerical issues, density as an artificial intermediate variable, or volume constraints. The UPM was utilised and programmed for linear elastic materials with isotropic and orthotropic properties, in earlier research.

In this work, we extend the UPM approach to hyperelastic materials, including material and geometrical nonlinearities not previously considered. For hyperelastic materials, we use the Neo-Hookean model (physical nonlinearity) in conjunction with Green-Lagrange strains as geometric nonlinearity (large amplitude deformation).

2 Methods

The procedure in the UPM method is the minimisation of the standard deviation of a pre-defined mechanical property that is chosen by the user, namely

$$\begin{cases} \min s_{\boldsymbol{H}}(\boldsymbol{H}) \\ st\ [k]\{u\} = \{f\} \\ \alpha_{min} \le \alpha_j \le \alpha_{max},\ \ j = 1,2,3, \dots . N \end{cases} \tag{1}$$

in which s_H stands for the standard deviation of $\boldsymbol{H}$; $[k]$ is the stiffness matrix, $\{u\}$ is the nodal displacmenet vector, and $\{f\}$ is the nodal force vector; α_j refers to the mechanical variable of each element that in total will be N elements in the volume, and it is limited between a selected minimum and maximum variable. In Equation. (1), $\boldsymbol{H}$ stands for the sensitivity of the strain energy with respect to the mechanical variable. For example in the linear elastic materials, it is given as $\boldsymbol{H} = \partial W/\partial E$ where W is the strain energy function.
Using an iteration loop the optimised structure based in the UPM is obtained with the following update equation for the mechanical variable

$$\alpha_i{}^{t+1} = \alpha_i{}^{t}\left(1 + \frac{H_i{}^{t} - \bar{H}^{t}}{s_H{}^{t} * K}\right) \tag{2}$$

where $\bar{H}$ is the mean of vector $\boldsymbol{H}$ and K stands for a modulated parameter of the magnitude of the step performed.
The formulation expressed above is the general trend in the UPM that can be implemented for linear and nonlinear elastic materials. The main difference between linear and nonlinear case is the shape of the strain energy function as well as the strains. In what follows we present the equations for both types of material.

2. 1 Linear elastic materials

For the elastic materials the strain energy is expressed as

$$W = \left\{\int_{\Omega} \frac{1}{2}\varepsilon : \mathbb{C} : \varepsilon\right\} d\Omega \tag{3}$$

where ε is the small strain and $\mathbb{C}$ is the material stiffness tensor wherein the mechanical variables are present. For the linear elastic materials these mechanical variables can be Poisson's ratio or Young's modulus, which in the previously published papers latter has been selected as the mechanical variable.

2.2 Nonlinear elastic material

Here we show the equations for the nonlinear elastic materials. The first strain energy function is Sent Venant that is just an extension of the linear elastic materials, given as

$$W = \int_{\Omega} \left\{\frac{1}{2}\lambda(tr\,\boldsymbol{E})^2 + \mu\boldsymbol{E}:\boldsymbol{E}\right\} d\Omega \tag{4}$$

where $\boldsymbol{E}$ is the Green-Lagrange strain tensor for considering the geometric nonlinearity, μ and λ are Lame's constants that are formulated as

$$\lambda = \frac{\nu E}{(1+\nu)(1-2\nu)} \qquad \mu = \frac{E}{2(1+\nu)} \tag{5}$$

where E and ν are Young's modulus and Poisson's ratio respectively. For the Saint Venant the mechanical variables can be Young's modulus and Poisson's ratio. If the mechanical variable is Young's modulus, the parameter $\boldsymbol{H}$ can be derived from Equation. (4) as follows

$$H = \int_{\Omega} \left\{\frac{1}{2}R_1(tr\,\boldsymbol{E})^2 + R_2\,\boldsymbol{E}:\boldsymbol{E}\right\} d\Omega \tag{6}$$

in which

$$R_1 = \frac{\nu}{(1+\nu)(1-2\nu)} \qquad R_2 = \frac{1}{2(1+\nu)} \tag{7}$$

Another model is the neo-Hookean that we formulate here. We consider a compressible neo-Hookean strain energy function, for example

$$W = \int_{\Omega} \left\{\mu(I_1 - 3) - \mu \ln J + \frac{\lambda}{2}(\ln J)^2\right\} d\Omega \tag{8}$$

where μ and λ stand for Lame's constants given in Equation. (5), I_1 is the first invariant of the right Cauchy-Green deformation tensor and J is the determinant of the deformation gradient tensor. For the neo-Hookean model we have also two mechanical variables the same as Saint Venant strain energy function.

3 Algorithm and compute code for UPM

Based on the above equations, UPM-based topology optimisation can be used for different materials ranging from linear to non-linear. Here in Table. 1 shows the general algorithm for the UPM, which can be programmed in a suitable programming language such as MATLAB and Julia.

Table. 1 The UPM algorithm

Input: Required initial conditions and parameters.
do FEM analysis at iteration $t = 0$
Calculate → $H, \overline{H}, s_H$
Calculate → W at itertaion $t = 0$

$$A^t = \frac{W^t - W^{t-1}}{W^t}$$

while ($A^t > \epsilon$ or $t \leq 1$) **do**
    do FEM analysis at iteration $t + 1$
    Calculate → $H, \overline{H}, s_H$
    Calculate → W
    Update $\alpha_i{}^{t+1} = \alpha_i{}^t \left(1 + \frac{H_i{}^t - \overline{H}^t}{s_H{}^t * K}\right)$
    if $t > 0$ **then**
        Calculate → A
        if $A < \epsilon$ **then**
            The design is optimum: $\alpha_i{}^{t+1}$ for each element
        else

        $t = t + 1$

        end if
    end if

end while

4 Conclusions and Contributions

In this work we developed equations for the UPM based topology optimisation for the materials with material and geometrical nonlinearities. In the previous works, the UPM was primarily developed for the linear elastic materials and here we show that it can be easily extended for the nonlinear materials. We formulated equations for Sain Venant and neo-Hookean strain energy function. However, it can be for other hyperelastic strain energy. This method is robust and can be implemented for nonlinear materials with different properties like anisotropy.

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