Proceedings of the Fifteenth International Conference on
Computational Structures Technology
Edited by: P. Iványi, J. Kruis and B.H.V. Topping
Civil-Comp Conferences, Volume 9, Paper 3.8
Civil-Comp Press, Edinburgh, United Kingdom, 2024
ISSN: 2753-3239, doi: 10.4203/ccc.9.3.8

Interactive Design Under the Multi-Framework of Topology Optimization with Human Intervention

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Abstract

This paper proposes a topology optimization method under a multi-framework approach that allows for human intervention. The objective is to leverage the rich experience and expertise of designers to dynamically adjust and control the optimization process, thereby yielding faster and more practical design solutions. To achieve this aim, an interaction strategy is developed between the Solid Isotropic Material with Penalization (SIMP) method, which employs an implicit geometric description, and the Moving Morphable Components/Voids (MMC/MMV) approach, characterized by an explicit geometric representation. Proposed method incorporates structural components into the pixel-based SIMP method, enabling intuitive control over the topology and geometric parameters of a structure. Through direct manipulation of these structural components, designers can add or adjust critical components, remove or modify redundant components, improving structural performance. Human intervention facilitates a richer and more diverse range of design options to meet specific application scenarios and performance criteria. The interaction across multiple methods also allows the direct importation of SIMP-based optimization results into Computer-Aided Design (CAD) software, enhancing design flexibility and convenience. The effectiveness of the proposed method is validated through multiple standard examples.

Keywords: solid isotropic material with penalization, moving morphable components/voids, multi-framework, human intervention, topology optimization, structural components.

1 Introduction

Topology optimization has become an integral part of modern design methodologies. Its primary advantage lies in its ability to generate innovative and efficient design solutions that surpass traditional approaches in achieving lightweight structures without relying on manual experience. Currently, topology optimization has reached a high degree of automation, requiring only engineers to assess the quality of the final results. It has been extensively applied in the manufacturing of industrial equipment for the aerospace, automotive, and shipbuilding sectors.

However, variations in operational conditions or adjustments made to simplify the manufacturing process can impact the optimized structure's physical performance [1]. To align more closely with real-world application scenarios, these issues can be addressed by increasing the complexity of the mechanical models or constraints, such as incorporating optimizations with stress constraints [2-6] and buckling constraints [7-10]. Enhancing design applicability with new variables increases optimization complexity, leading to longer computational times due to the iterative nature of topology optimization. Often, complex design problems are not directly solvable, requiring designers to make extensive adjustments, turning topology optimization into a trial-and-error process. This contradicts its intended use in structural design, presenting a challenge to balance usability and innovation while efficiently incorporating design requirements into topology optimization.

Some studies have proposed human-machine interactive frameworks that merge the exploratory capabilities of fully automated topology optimization with the professional knowledge of human design engineers. The objective is to expand the application of topology optimization to industrial applications currently deemed too time-consuming and computationally demanding. This new framework is characterized by the dual incorporation of automated machine discovery and rich human experience, synergizing to elevate satisfaction with the design quality [11]. In addition, researchers have explored applying evolutionary methods to truss design and continuous topology optimization, as evidenced by the studies of Mueller and Ochsendorf [12] and Yang et al. [13]. These methods produce several design solutions with similar performance levels, allowing designers to select the one that matches their aesthetic preferences. The human-informed topology optimization (HiTOP) algorithm proposed by Ha and Carstensen [14] utilizes an interactive scheme. In this approach, design decisions are co-guided by human and machine efforts. An initial exploratory design iteration is conducted using a standard topology optimization process. After a number of initial iterations, the program is paused for the design engineer to intervene. They use an elliptical shape to define regions of interest (ROI) and implement minimal characteristic dimension control in those local areas to improve structural performance.

The SIMP (Solid Isotropic Material with Penalization) method in structural optimization has gained popularity for its applicability and flexibility but faces challenges in incorporating human intervention during topology optimization. The method produces results as a density distribution, which lacks clear interpretability and does not straightforwardly present structural features like sizes and void patterns. This complicates intuitive understanding and direct manipulation of the design, requiring post-processing for clearer geometric features, thus increasing design complexity and limiting the SIMP method's intervenability.

This study proposes an interactive framework for structural optimization. This method facilitates human-intervened design for the SIMP approach, allowing real-time modifications to the current structure to speed up convergence or meet design requirements. The crux of the current method lies in achieving a phased strategy for mutual conversion between the SIMP approach and explicit topology optimization methods, such as Moving Morphable Components/Voids (MMC/MMV) [18-21], transforming pixel-based optimization results into more intuitive, maneuverable structural element descriptions. Within this framework, engineers can directly control sizes, thicknesses, and the addition or removal of components, leading to more flexible and convenient design adjustments and enhancing the usability of the final design. Geometric transformations from pixel to precise parameters enhance mechanical control, improving structural stability. Human-centered design methods increase manufacturability and design flexibility. By integrating SIMP with CAD, the approach streamlines digital processes, reducing the need for post-processing.

The arrangement of this paper is as follows. Section 2 and 3 provides a detailed introduction to the Solid Isotropic Material with Penalization (SIMP) topology optimization method and its explicit conversion mechanism to results based on Moving Morphable Components (MMC). Section 4 focuses on showcasing a series of interactive optimization examples. These examples demonstrate the application of the proposed method and verify its practical effectiveness through specific case studies. Finally, Section 5 concludes the paper, summarizing the research findings, discussing the significance and limitations of the study, as well as future research directions.

2 Methods

2.1 The SIMP Topology Optimization Framework

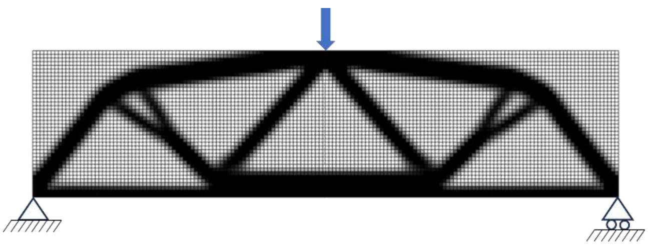


Figure 1: Classical SIMP Method Example with MBB Beam

In classical topology optimization problems, the minimization of compliance primarily focuses on optimizing the distribution of material to obtain a structure with maximum stiffness under given load conditions and volume constraints. As illustrated in Figure 1, to accurately evaluate structural performance, the design domain is divided into several elements. Within the SIMP framework, the state of e-th element, is defined by its density ρ_e, where $\rho_e = 1$ represents a solid element, and $\rho_e = 0$ indicates the absence of material. The specific optimization formulation can be expressed as follows:

$$\begin{aligned} Find\ \ & \boldsymbol{\rho}^{\top} \\ Min\ \ & \boldsymbol{f}^{T}\boldsymbol{u} \\ S.t.\ \ & \left(\sum_{e=1}^{N} x_e^p \boldsymbol{K}_e\right)\boldsymbol{u} = \boldsymbol{f}, \\ & \sum_{e=1}^{N} v_e \rho_e \leq V\,, \\ & 0 < \rho_{min} \leq \rho_e \leq 1, e = 1, \ldots, N \end{aligned} \tag{1}$$

The essence of the optimization problem lies in minimizing the objective function $C = \boldsymbol{f}^T\boldsymbol{u}$, where the global stiffness matrix $\boldsymbol{K}$ and the global displacement vector $\boldsymbol{U}$ satisfy the static equilibrium equation $\boldsymbol{Ku} = \boldsymbol{f}$, with $\boldsymbol{f}$ representing the global load vector applied to the structure. Moreover, a volume constraint is introduced to limit the use of material, where V denotes the upper limit of the volume fraction, and v_e represents the volume of each element. p is the penalization factor, and ρ_{min} refers to a minimum density limit.

Despite the widespread attention the SIMP method has garnered among researchers, the challenges it faces in facilitating human intervention cannot be overlooked. Given that the optimization results produced by the SIMP method are predominantly presented in the form of density distributions, lacking intuitive geometric features, designers find it difficult to make direct, targeted modifications to structural features.

2.2 The MMC Topology Optimization Framework

In contrast to the SIMP method, the MMC approach utilizes components as the fundamental elements to describe the structure's topology, with the corresponding geometric parameters of these components serving as the design variables for optimization. This component-based approach to structural topology description provides more intuitive geometric information for human intervention, allowing designers to modify the topology layout using components as the basic operational entities. Figure 2 illustrates the fundamental concept of this method. Under the MMC framework, the topology of a structure can be expressed as follows:

$$\begin{cases} \phi^a(\boldsymbol{x}) > 0, if\ \boldsymbol{x} \in \Omega^a \\ \phi^a(\boldsymbol{x}) = 0, if\ \boldsymbol{x} \in \partial\Omega^a \\ \phi^a(\boldsymbol{x}) < 0, if\ \boldsymbol{x} \in D \backslash \Omega^a \backslash \partial\Omega^a \end{cases} \tag{2}$$

In this framework, $\boldsymbol{D}$ represents the design domain, while Ω^a denotes the region occupied by all solid components. ϕ^a represents the topology description function, with $\phi^a = \max(\phi^1, \phi^2, \ldots, \phi^i)$, where ϕ^i is the Topology Description Function (TDF) of the i-th component.

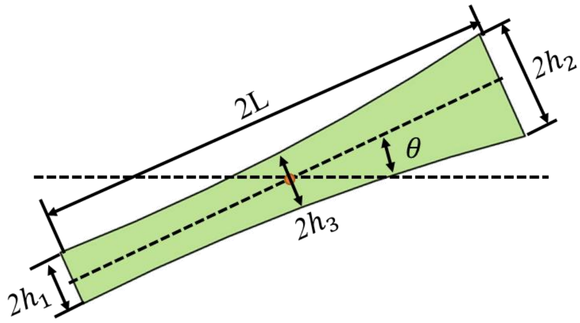


Figure 2: Component with Quadratically Variable Thickness

This study adopts quadratic thickness components, as shown in Figure 2, with the TDF defined as follows:

$$\phi_i(x, y) = \left(\frac{x'}{L_i}\right)^c + \left(\frac{y'}{f(x')}\right)^c - 1 \tag{3}$$

$$\begin{Bmatrix} x' \\ y' \end{Bmatrix} = \begin{bmatrix} cos\theta_i & sin\theta_i \\ -sin\theta_i & cos\theta_i \end{bmatrix} \begin{Bmatrix} x - x_{0i} \\ y - y_{0i} \end{Bmatrix} \tag{4}$$

In the TDF function, c is a large even number, typically chosen as 6, with larger values resulting in sharper corners at the component junctions. x_{0i} and y_{0i} represent the center coordinates of the component, while L_i represents half the length of the component. θ_i denotes the angle from the horizontal direction to the component's centerline (measured counterclockwise). For quadratic thickness components, $f(x')$ is expressed as:

$$f\left(x^{'}\right) = \frac{h_1 + h_2 - 2h_3}{2(L)^2}(x')^2 + \frac{h_2 - h_1}{2L}x' + h_3 \tag{5}$$

For numerical processing, a regularized version of the Heaviside function is typically used in place of the traditional Heaviside function [22], converting TDF values into density values between 0 and 1. In this study, $H_\epsilon(x)$ is employed as the regularized form of the Heaviside function:

$$H_\epsilon(x) = \begin{cases} 1, & if\ x > \epsilon \\ \frac{3(1-\alpha)}{4}\left(\frac{x}{\epsilon} - \frac{x^3}{3\epsilon^3}\right) + \frac{1+\alpha}{2}, & if - \epsilon \le x \le \epsilon \\ \alpha, & otherwise \end{cases} \tag{6}$$

Like many other topology optimization methods, the MMC approach utilizes four-node bilinear elements to discretize the design domain uniformly. The density of an element can be determined through interpolation methods:

$$\rho^e = \frac{\sum_{i=1}^{4}\left(H_\epsilon(\phi_i^e)\right)^q}{4} \tag{7}$$

Here, $H = H_\epsilon(x)$ represents the Heaviside function, and $\phi_i^e (i = 1, \dots, 4)$ are the TDF values at the four nodes of e-th element. ϵ is the parameter controlling the regularization magnitude, and α is a small positive number used to ensure the non-singularity of the stiffness matrix in calculations. q acts as a density penalization factor, similar to the one used in the SIMP method.

3 Numerical Implementation of Human-Intervened Topology Optimization

3.1 Interaction Conversion Strategy Between Explicit/Implicit Topology Optimization Models

This paper proposes a phased strategy for the automatic identification of structural members from implicit topology optimization results. This method retains the structural features found in SIMP optimization results, ensuring a natural transition between structural elements. Thus, when designers intervene in the subsequent optimization process, they can adjust structural features more accurately. To achieve this, we utilize the concept of components from MMC, defining structural members as composed of components (including intersections of multiple components). The key lies in how to transform implicit topology optimization results into explicit topology optimization results described based on components. Figure 3 demonstrates the specific fitting process from SIMP results to component layouts. Initially, SIMP's density distribution is converted into a topology skeleton using image processing, where pixels above a threshold are set to 1 (others to 0) for binarization. MATLAB's bwmorph algorithm skeletons the density field, from which endpoints and branches are thickened for clarity, then removed for component fitting. After diagonal filling and cleaning to refine the skeleton, the regionprops function calculates geometric properties (centroid, axis lengths, orientation) for each region. These properties guide the fitting of explicit topology structures with rectangular components, forming a structural array that outlines component layouts for further design and analysis.

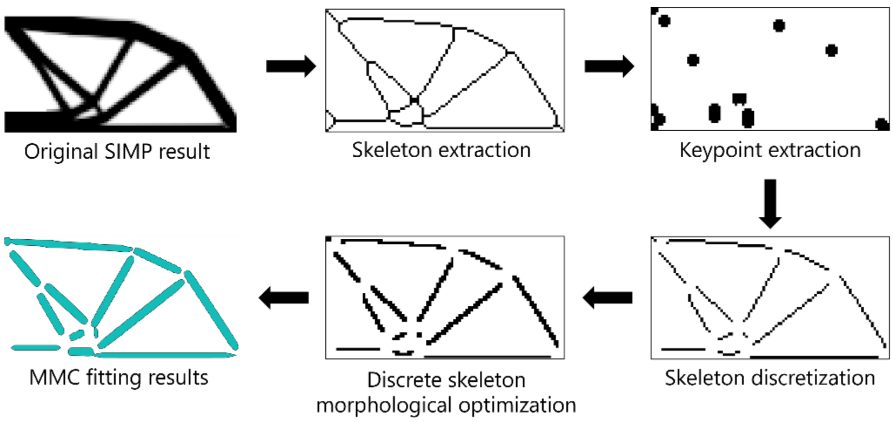


Figure 3: MMC Fitting Process of SIMP Results

As illustrated in Figure 3, the preliminary component layout clearly presents the topological configuration of the SIMP optimization results. Subsequently, based on this initial configuration, we achieve a precise implicit-to-explicit model transformation by solving the following optimization problem, which is formulated as:

$$\begin{aligned} &Find \quad \boldsymbol{D} = (\boldsymbol{D}^1, \boldsymbol{D}^2, \dots, \boldsymbol{D}^n) \\ &Min \quad C = \int_D (\boldsymbol{\rho}_s - \boldsymbol{\rho}_c)^2 d\Omega \end{aligned} \tag{8}$$

Here, $\boldsymbol{D}$ represents the design variables of the components, and n denotes the total number of existing components in the initial structure. $\boldsymbol{\rho}_s$ and $\boldsymbol{\rho}_c$ respectively represent the density distribution of the implicit and explicit models. Furthermore, the sensitivity of the objective function to the design variables can be expressed as:

$$\begin{aligned} \frac{\partial C}{\partial d} &= \int_D 2(\rho_c^{\mathrm{e}} - \rho_s^e) \frac{\partial \rho_c}{\partial d} d\Omega \\ &= \int_D 2(\rho_m^{\mathrm{e}} - \rho_s^e) \sum_{i=1}^{4} \frac{q}{4} \left(\frac{\partial H_\epsilon(\phi_i^e)}{\partial d} \right)^{q-1} d\Omega \\ &= \frac{q}{2} \int_D (\rho_m^{\mathrm{e}} - \rho_s^e) \sum_{i=1}^{4} \left(\frac{\partial H_\epsilon(\phi_i^e)}{\partial d} \right)^{q-1} d\Omega \end{aligned} \tag{9}$$

where $\rho_c^{\mathrm{e}}, \rho_s^e$ refer to the elemental densities of the explicit and implicit models, respectively. $\partial H_\epsilon(\phi_i^e)/\partial d$ is obtained through differential or analytical methods, detailed in the referenced literature [18,19].

To transform an explicit geometric model into an implicit model, replacing $\boldsymbol{\rho}_c$ with $\boldsymbol{\rho}_s$ suffices. The entire transformation process is detailed in Figure 4.

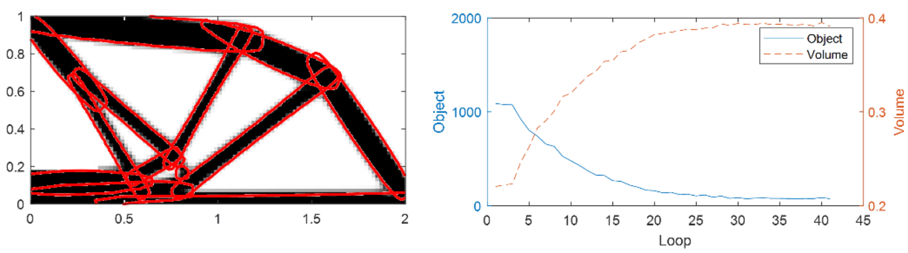


Figure 4: Final MMC Fitting Outcome

For converting an implicit model into an explicit model described by voids (e.g., voids described using closed B-spline curves), one can skip the initial component construction step and proceed directly to void fitting, updating Eq. (10) as follows:

$$
\begin{aligned}
&Find \quad \boldsymbol{B} = (\boldsymbol{B}^1, \boldsymbol{B}^2, \dots, \boldsymbol{B}^n) \\
&Min \quad C = \int_D (\boldsymbol{\rho}_s - \boldsymbol{\rho}_c)^2 d\Omega
\end{aligned} \tag{10}
$$

Here, $\boldsymbol{B}$ represents the design variables for the voids, and n denotes the total number of voids used. The reason for not requiring an initial configuration when using void descriptions is due to the superior deformation capability of the B-spline-based void descriptions compared to component descriptions. Thus, even if the explicit description's initial configuration is far from the final one, an optimal void layout can be quickly achieved. The analytical sensitivities for Eqs. (8) and (9) are straightforward and not elaborated upon here.

3.2 Interactive Method

Within the current optimization framework, engineers are allowed to intuitively participate in the design optimization process (at which point the optimization is paused) to provide more precise guidance for subsequent optimizations. Once the interactive phase concludes, the optimization program resumes. Notably, subsequent optimizations can be conducted either within the SIMP framework or directly on the MMC framework, as we possess both explicit and implicit optimization models. Below, we detail the technical nuances of conducting structural topology updates based on the needs of manual intervention.

3.2.1 Component Addition Method

In certain scenarios, to enhance the performance of specific areas within a structure, it becomes necessary to add components at particular locations. Furthermore, components with specific layouts may offer better performance characteristics, necessitating the replacement of existing designs with components of specific shapes. Engineers can adjust an elliptical-like shape by dragging, thereby obtaining two endpoints of the quasi-ellipse, $(x_1, y_1), (x_2, y_2)$. Based on these points, key parameters for the new component are calculated according to Eq. (11), including its length l, center coordinates (x_0, y_0), and its angle θ relative to the horizontal line, to construct a new component through these points. Thickness does not need to be calculated; it can be directly obtained by adjusting the length of the short axis of an ellipse.

$$
\begin{gathered}
l = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2} \\
(x_0, y_0) = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right) \\
\theta = atan\left(\frac{y_2 - y_1}{x_2 - x_1}\right)
\end{gathered} \tag{11}
$$

The added component will be optimized within SIMP as a non-designable solid domain. If subsequent optimizations are conducted under the MMC framework, corresponding adjustments to parameters within MMA are required, including expansions to the upper and lower bound vectors.

3.2.2 Component Removal and Adjustment Method

In some cases, to improve product reliability and accommodate manufacturing capabilities, simplification of the design is necessary, especially by eliminating unnecessary parts and interfaces. This involves the removal of specific structural components from certain areas of the structure. We propose an interactive mode-based component removal method. Initially, a Topological Description Function (TDF) is used to determine whether a point clicked by the engineer belongs to a specific component. If the value of $\phi^i(\boldsymbol{x})$ for a component at the clicked point's coordinates is greater than 0, it is considered to belong to the i-th component; otherwise, it does not.

$$\begin{cases} \phi^i(\boldsymbol{x}) > 0, if\ \boldsymbol{x} \in \Omega^i \\ \phi^i(\boldsymbol{x}) = 0, if\ \boldsymbol{x} \in \partial\Omega^i \\ \phi^i(\boldsymbol{x}) < 0, if\ \boldsymbol{x} \in D\backslash\Omega^i\backslash\partial\Omega^i \end{cases} \tag{12}$$

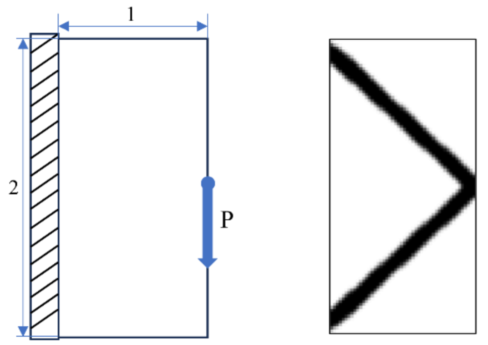
Component removal is achieved by either removing the corresponding variables from the design variable vector or setting the component's variable value to zero, i.e., $\boldsymbol{D}^i = 0$. If subsequent optimizations are conducted under the MMC framework, corresponding parameters in MMA must be appropriately removed or adjusted. Removed components will be optimized in SIMP as non-material non-designable domains. By comparing the density field before and after interaction, only the changed parts are set as non-designable domains to avoid affecting overlapping areas of components.

Using a similar method, the corresponding component can be identified, allowing for the modification and editing of relevant dimensions to achieve adjustments to the structural elements.

4 Numerical Examples

4.1 Structural Buckling Optimization Example

In the domain of structural optimization, enhancing buckling performance presents a significant and challenging task. This section elaborates on effectively improving a structure's buckling performance through a canonical example of a short cantilever beam. As depicted in Figure 5a, the design domain measures 1×2, with the left boundary fixed and a concentrated load P=**1** applied at the mid-point of the right side. The design domain is discretized into 50×100 elements with a volume constraint of 0.15. Unless otherwise specified, all examples utilize the same material properties, with the elastic modulus E set to 1 and Poisson's ratio v set to 0.3.

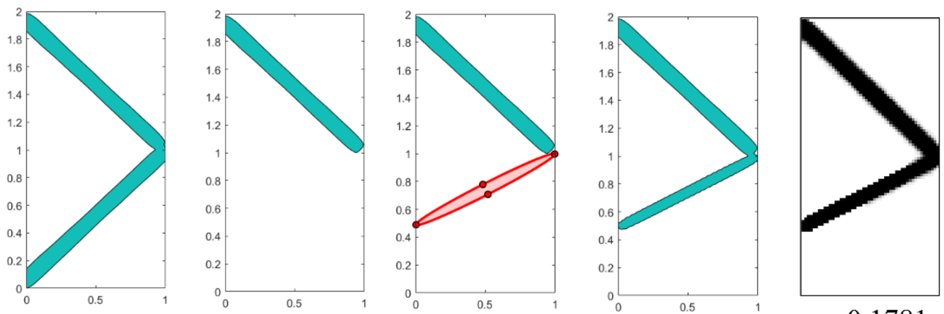


c=0.1709
BLF=0.0367

(a) (b)

Figure 5: Short Cantilever Beam Design Problem and Its Compliance Optimization Results

Figure 5b shows the optimization result achieved by minimizing compliance under the same design domain, boundary conditions, and load scenario. The structure consists of two uniformly thick bars distributed above and below. From the perspective of compliance optimization alone, this structure is optimal; however, its buckling performance is relatively weak. Considering the direction of the applied load, the bottom bar bears compression. Enhancing the structure's buckling performance can be effectively achieved by adjusting the thickness of the bottom bar or changing its position to alter its length.



Translation Deletion Addition Result c=0.1781 BLF=0.0437

(a) (b) (c) (d) (b)

Figure 6: Manual Intervention Operations and Final Compliance Optimization Results

Optimization of the same example is carried out by adopting the proposed optimization strategy. In this case, adjustments are made to the existing design's bottom bar, removing it and relocating it slightly higher. The adjusted optimization result, as shown in Figure 6, reduces the length of the bottom bar and increases its angle relative to the direction of the applied load, effectively reducing the stress on the bar and enhancing overall buckling performance.

Notably, despite these adjustments, the structure's compliance was almost unaffected, only increasing by 4.21%, while the Buckling Load Factor saw an improvement of 19.09%. This result indicates that although the adjusted structure did not achieve the optimal state of buckling performance, it managed to enhance a certain aspect of the structure's performance in a rapid manner.

Traditional buckling optimization methods, even when calculating only the first eigenvalue (i.e., $neig = 1$) and setting the eigenvalue calculation tolerance to tol =1e-3, still require more than 5 minutes to solve. However, with the optimization strategy proposed in this study, even including manual operation time, the entire process takes less than 1 minute, significantly improving efficiency. It is noteworthy that as the problem scale increases and the number of design variables in the design domain grows, this time difference will become even more pronounced.

4.2 Rich Design Generation

This study introduces an innovative interactive structural design approach aimed at generating diverse structural distributions during the design process, akin to the working mechanism of Generative Adversarial Networks (GAN). However, the uniqueness of this method lies in its independence from the complex training processes required by GANs, relying instead on direct designer interaction to set various initial distribution parameters, guiding the diversity of structural designs.

Specifically, designers can easily set initial distribution parameters through an intuitive interactive interface, serving as the basis for generating structural samples. A significant advantage of this method is its ability to rapidly produce a series of diverse design solutions in a short time, eliminating the need for prolonged training associated with GAN models. Moreover, the structures generated by this method possess more practical physical significance and engineering applicability compared to those generated by GANs, as they represent local optimal solutions for specific optimization problems.

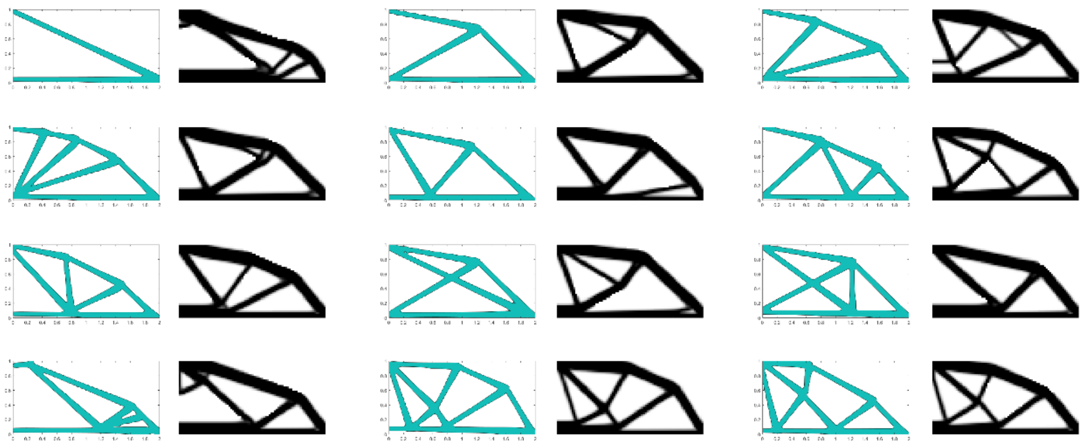


Figure 7: Example of Interactive SIMP Optimization Design Based on Movable MMC Components

Figure 7 showcases the interactive-guided structural topology optimization process using the MMC technique, conducted through interactive guidance. The optimization case is the MBB beam problem; given the structure's symmetry, only half of the design domain is displayed, measuring 2×1. In this study, to enhance design space diversity and explore its potential, MMC technology is introduced to set initial distributions before optimization, allowing users to guide the formation of initial distributions through interactive operations, thus influencing the final optimization results. The Figure 7 presents 12 unique design solutions. In each subplot, the left side shows the structure layout under MMC interaction, while the right side reveals the results of structural optimization conducted using the SIMP method guided by this layout. Except for the first design, with a compliance of 106.51, the compliance values for the remaining designs range from 97.98 to 101.92.

5 Conclusions and Contributions

This paper successfully constructs and implements an innovative topology optimization framework that extensively expands upon the traditional density-based compliance minimization approach, ingeniously integrating elements of human-computer interaction. This unique design philosophy fully leverages the expertise and extensive experience of design engineers, offering them greater autonomy in the design process. Engineers can proactively adjust specific features of the structure based on actual needs, effectively seeking the optimal balance between complex performance requirements and manufacturing constraints. The key innovation of this study lies in the introduction of the MMC concept, which transforms the results generated by the standard SIMP method into a more intuitive and manageable rod-like structural form. This transformation not only enhances the flexibility and practicality of the design but also makes the final design results more coherent.

Through the application analysis of examples, the effectiveness of this framework in improving design efficiency and reducing computational resource consumption is fully demonstrated. Especially in daily and on-site design scenarios, this framework shows its unique advantages. However, it should be clarified that although this framework provides an efficient design strategy, it is not intended to completely replace traditional, more complex topology optimization methods. For scenarios capable of utilizing high-performance computing resources, traditional methods remain the preferred choice.

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