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A Solution Strategy for Robust Topology Optimization when the Nominal and Expected Compliance are the Same

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Abstract

In this paper, an anomaly resolving strategy is presented for such cases where the nominal and robust compliances are the same in the volume-constrained continuous topology optimization with directionally uncertain loads. In this investigation, the parametric normally and uniformly distributed expected compliance and the non-parametric total-compliance-variance are used as robustness measures. It will be proved by experimental evidence, that in the anomalous cases the result of the nominal-compliance minimization and the results of the minimization of the investigated robust measures form a pareto optimal point, therefore the volume-percentage increasing is the only way of the design robustness improving.

Keywords: robust, topology, volume-constrained, uncertain loads.

1 Introduction

Uncertainty is one of the important aspects of continuous topology optimization to create robust and reliable solutions. There are several ways to take uncertainty into account in topology optimization of continuous structures, which can be distinguished according to the structural and design features that induce uncertainty.

In this paper, it is assumed that the only source of uncertainty is the variability of the applied load directions, where the uncertainty of the load direction is represented by a symmetric angle defined around the nominal direction. Most models in this area use parametric statistical tools to describe the directional uncertainty of applied loads to produce robust structures that are as insensitive as possible to directional uncertainty (e.g., Chen et.al [1], Dunning et.al [2], Alvarez and Carrasco [3], Guest and Igusa [4], Conti et.al [5], and Schuëller and Jensen [6]. In the most popular parametric statistical approaches, the preferred measure of robustness is the expected compliance with a normal distribution.

In a recent study (see in Csébfalvi and Lógó [7]), it was shown that the expected compliance is not a universally applicable robustness measure, because it can give misleading results in some non-symmetrical cases.

In the research field of robust optimization, robust and deterministic optimal configurations are generally expected to differ. Therefore, most research in this area aims to illustrate the effects of uncertainty with a comparison of the robust and nominal (deterministic) designs. The problem with this approach is that it is easy to define such a non-symmetrical case (see Rozvany [8]) in which the robust expected-compliance minimum and the nominal-compliance minimum design will be the same. It should be noted, however, that the importance of taking uncertainty into account cannot be reduced even if the robust and nominal design are identical. This anomaly requires new techniques that allow for deeper insights. The aim of this study to propose a strategy to tackle generally the effects of uncertainty, which could be apply even in anomaly cases pointed out above.

In this paper, an anomaly-resolving strategy is presented for such cases when the nominal and robust compliance are the same in the optimization of a volume-constrained continuous topology with uncertain direction of loads. It will be presented with experimental evidence that in the anomalous cases, the results of minimizing nominal compliance and minimizing the robustness measures under consideration form a Pareto-optimal point, and therefore increasing the volume-percentage is the only way to improve the design robustness.

2 Methods

In this paper, the only source of uncertainty is the variability of the applied load directions. There are several deterministic and stochastic approaches that attempt to cope with this problem in structural design to produce robust structures that are as insensitive as possible to changes in applied load directions.

In this study, a normally distributed expected compliance ($\boldsymbol{expc}$), the rarely investigated uniformly distributed expected compliance ($\boldsymbol{unic}$), and the currently developed by Csébfalvi [9] non-parametric total-compliance-variance ($\boldsymbol{tvrc}$) are used as robustness measures.

It will be shown, that in the anomalous cases the result of the nominal-compliance minimization and the results of the minimization of the investigated robustness measures form a so-called Pareto-point, therefore the volume-percentage increasing is the only way of the design robustness improving. The SIMP-type algorithms were implemented in MATLAB in a common frame. In the case of parametric-measures ($\boldsymbol{expc}, \boldsymbol{unic}$) the Optimality Method (OC) was used to solve the measure-minimization problems. The total-compliance-variance minimization problem with constrained volume-percentage increase was solved by a constrained-nonlinear-minimization algorithm (**`fmincon)`**.

In Figure 1 and Figure 2 it is shown that what are the fundamental differences in the robust design searching process in the popular symmetric and the practically non-investigated non-symmetric cases. Because in the non-symmetric cases the robustness measures and the nominal-compliance form a pareto optimal point there is only one way to get a more robust design and it is the material fraction increasing.

In Figure 3 and Figure 4 the robust $\boldsymbol{tvrc}$ solution of a popular symmetric problem is shown with the compliance function shapes given by the minimization of the nominal- compliance ($\boldsymbol{nomc}$) and the ($\boldsymbol{expc}, \boldsymbol{unic}, \boldsymbol{tvrc}$) robustness measures

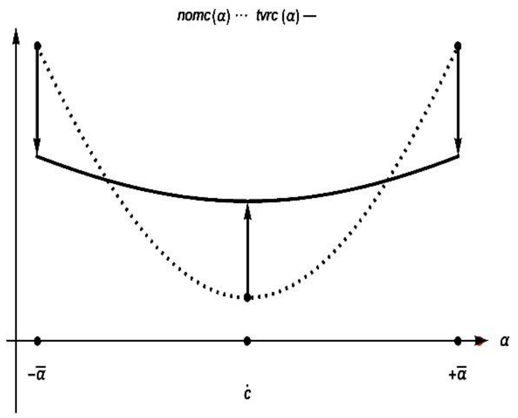

Figure 1: Solution strategy in normal case.

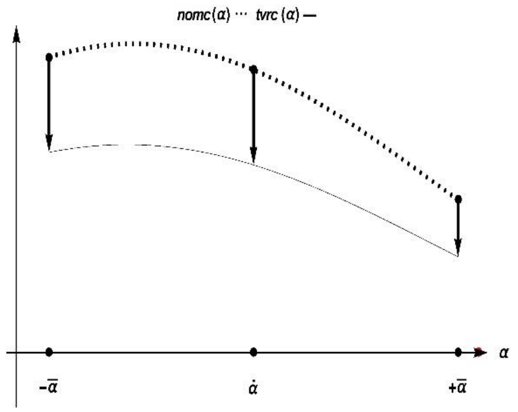

Figure 2: Anomalous solution strategy.

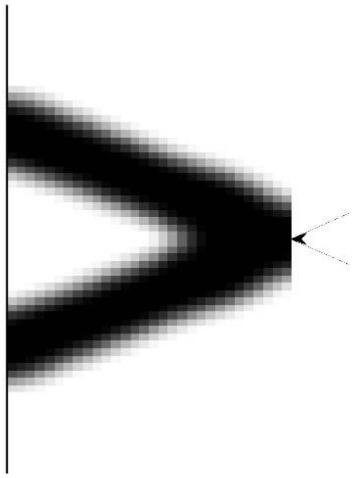

Figure 3: A robust symmetric design.

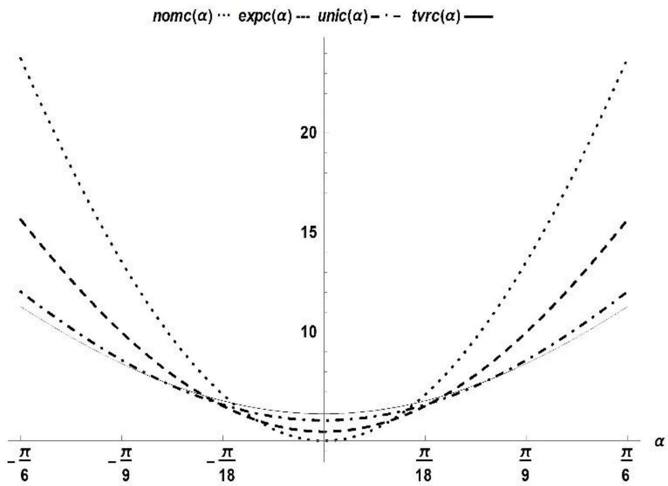

Figure 4: Symmetric measure shapes.

During the $\boldsymbol{tvrc}$ minimisation 30% nominal-compliance increment was allowed as a design parameter. In the case of the $\boldsymbol{unic}$ $(\boldsymbol{expc})$ minimisation the corresponding response variable was 20% (10%) with clearly detectable larger variability.

3 Results

The results obtained by the proposed method are presented for two load variants, one-dimensional and two-dimensional cases.

In the first case a concentrated force was applied to the end joint of the structure, in the second case two downward concentrated forces were applied.

In Figure 5 and Figure 6, the nominal compliance (**nomc**) minimal designs are presented for two non-symmetric designs with one and two directionally uncertain loads and $vol = 0.25$ volume setting.

In Figure 7 and Figure 8, the nominal compliance ($\boldsymbol{nomc}$) minimal designs of the same examples are presented with $vol = 0.25 + 0.05 = 0.30$ setting to get more balanced designs characterized by the total variation ($\boldsymbol{tvrc}$) measure.

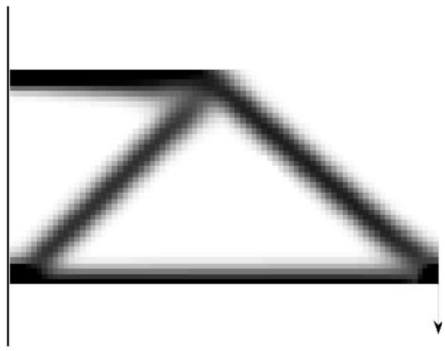

Figure 5: $\boldsymbol{nomc}$ minimal design with $vol = 0.25\%$ volume percent.

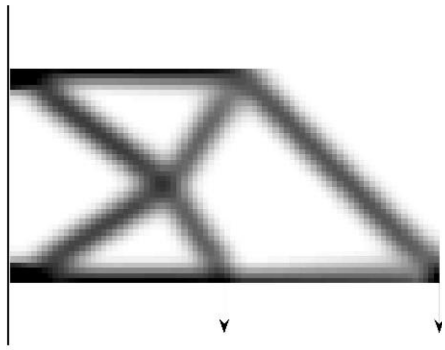

Figure 6: $\boldsymbol{nomc}$ minimal design with $vol = 0.25\%$ volume percent.

According to the behaviour of the investigated $\boldsymbol{nomc} = \boldsymbol{tvrc}$ “anomaly”, the 0.05% volume percentage increment drastically changes the compliance function shape in each case (see Figure 9 and Figure 10).

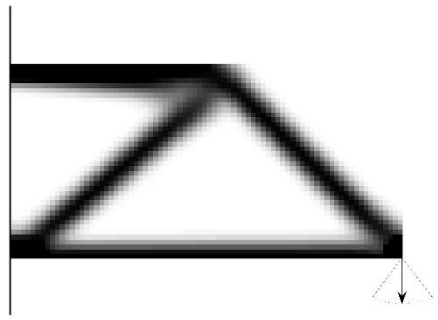

Figure 7: $\boldsymbol{nomc}$ minimal design with $vol = 0.30\%$ volume percent.

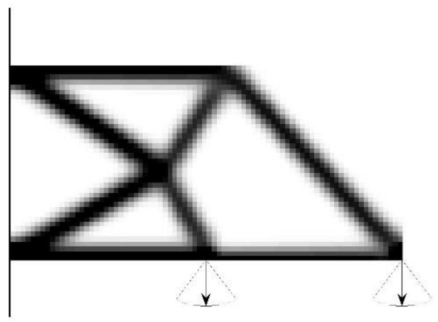

Figure 8: $\boldsymbol{nomc}$ minimal design with $vol = 0.30\%$ volume percent.

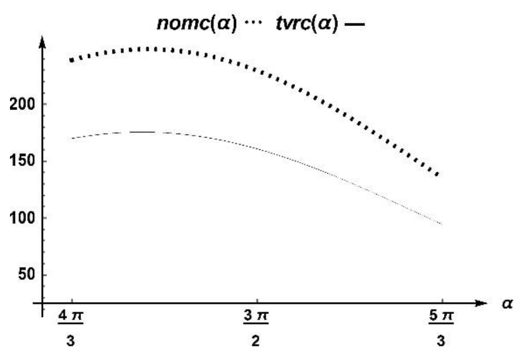

Figure 9: Common plot of $\{\boldsymbol{nomc}, \boldsymbol{tvrc}\}$ compliance curves.

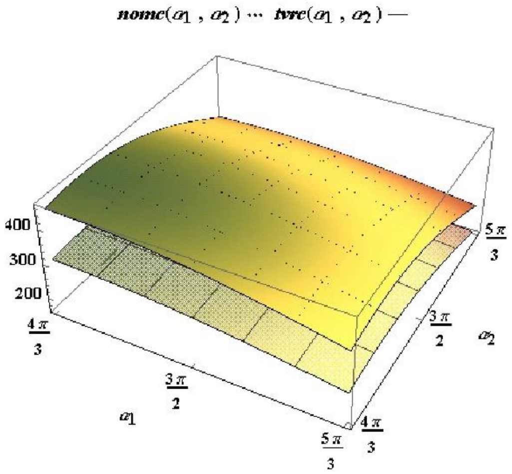

Figure 10: Common plot of $\{\boldsymbol{nomc}, \boldsymbol{tvrc}\}$ compliance areas.

In figures above it was demonstrated that using the terminology of the traditional variational analysis, the essence of the non-parametric robustness measure minimization is very simple: in the case of one directionally uncertain load it is a curve-length minimization problem, and in the case of two directionally uncertain loads a surface-area minimization problem has to be solved.

4 Conclusions and Contributions

In this paper, experimental evidences are presented to resolve the anomalous behaviour of the robust design searching process when solutions of the investigated robust approaches and the nominal approach form a pareto optimal point in the design space in the volume-constrained continuous topology optimization with directionally uncertain loads. The proposed methodology is based in a very simple and natural finding: in the "abnormal" case there is only way to increase the robustness of the design in a controllable form which is the volume fraction increasing. It is clear, that higher the allowed volume fraction increments the higher the chance to get a more balanced (more robust) design. The computational efficiency of the proposed approach in the anomalous case is extremely good, because to get a more robust design only the nominal compliance minimization process can be used. In this case, the different robustness measures (normally or uniformly distributed expected compliance and the total compliance variation) are only potential diagnostic tools which may help in the deeper understanding of the anomalous case. It is important to note, that the currently developed total compliance variation measure is the only one which has a controllable design parameter in the symmetric cases. It has to noted, that this feature is very important one from engineering point of view. Changing the allowed maximum nominal compliance increment we are able to select the best robust design. Using the terminology of the traditional variational analysis, the essence of the non-parametric robustness measure minimization is very simple: in the case of one directionally uncertain load it is a curve-length minimization problem, and in the case of two directionally uncertain loads a surface-area minimization problem has to be solved. Contrarily, in the cases of parametric normally or uniformly distributed stochastic measures the nominal compliance increment will be an uncontrollable response parameter which depends only on the more or less fictitious statistical assumptions without statistically correct validation (for example: normality validation) and parameters (for example: normally distributed standard deviation estimation). We hope that this work will contribute to a better understanding of what probabilistic and non-parametric robust optimization really means and it may be an inspirational starting point of further investigations in the future.

Acknowledgements

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